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Dispersive analysis of meson-meson scattering and light resonances

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GSI. Darmstadt. 20/11/2019

Motivation to study scattering of pions and kaons

- π,K are Goldstone Bosons of QCD → Test Chiral Symmetry Breaking

  - π,K appear as final products of almost all hadronic processes:
    Examples: B,D, decays, CP violation studies, etc…

- Light mesons not be the primordial interest of PANDA, but many processes with π's and K's in final state of D, D*, η_c decays… or in processes like $p\bar{p}$ →J/Ψ$\pi\pi, \pi\pi\eta, \pi K\bar{K}$, or D → πK, many three body decays with π's and K's, etc…

- Much debated scalar meson resonances appear, in particular the $f_0(500)$, $f_0(980)$, $f_0(1370)$, $f_0(1500)$ (in any channel with the lightest glueball) and also the $K^*_0(700)$

- Also interest in other waves, particularly the P-wave beyond the ρ(770)

- PANDA also claims to reach **precision**.

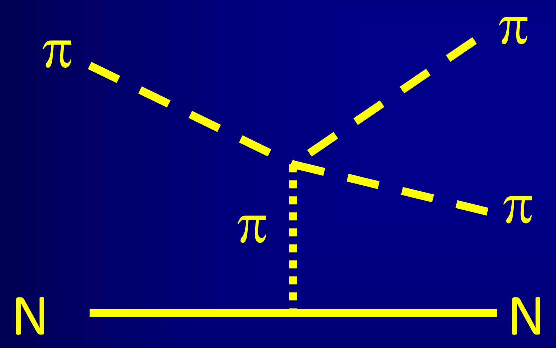
ππ and πK SCATTERING data are poor

- π and K are unstable. Still, beams can be made.

  But NOT luminous enough for **ππ and πK** collisions: <u>**Indirect measurements**</u>

1) From meson-Nucleon scattering

Chew-Low Extrapolation (see Gribov's book Sect. 2.6.2)



Initial state not well defined, model dependent off-shell extrapolations
(OPE, absorption, A_2 exchange...).
Needs Meson- N-partial wave extraction. Problems with phase shift ambiguities, etc...

As a consequence... VERY LARGE SYSTEMATIC UNCERTAINTIES

$\pi\pi$ scattering data. Example: CERN Munich Experiment

SYSTEMATIC uncertainties larger than STATISTICAL

Nuclear Physics B75 (1974) 189–245. North-Holland Publishing Company

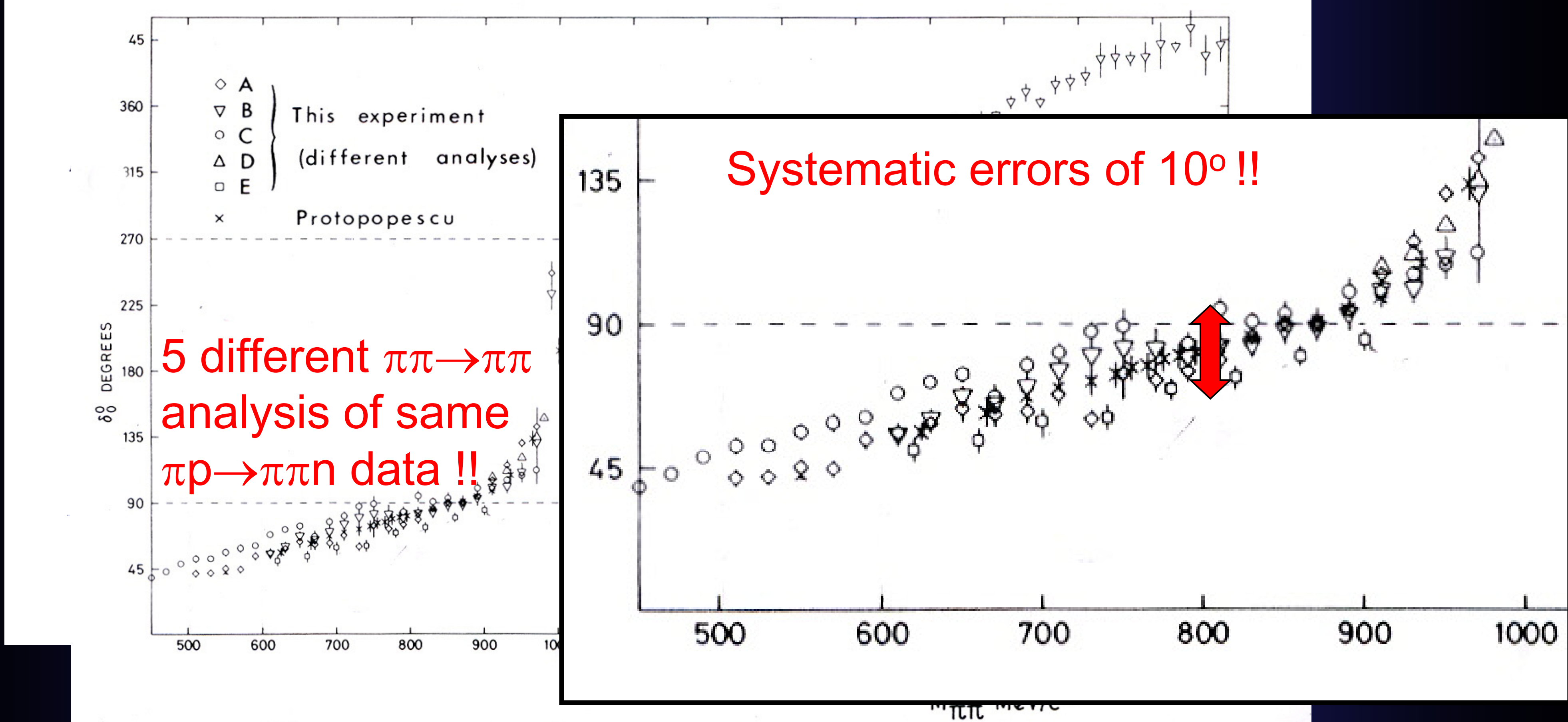


Fig. 31. The $\pi\pi$ scattering phase shift δ_0^0 for spin 0 and isospin 0 as determined by various analyses of our 17 GeV/c $\pi^+\pi^-$ data (A,B,C,D,E), compared with the previous results by Protopopescu et al. [28]: (A) Analysis based on pole extrapolations of the moments (subsect. 4.7.1. (A)). (B) Analysis at each $m_{\pi\pi}$ with $\pi^+\pi^-$ amplitudes assumed to be nucleon-spin and $\pi\pi$-spin coherent, involving a parametrization to describe the $m_{\pi\pi}$ dependence (subsect. 4.7.2. (B)). (C) Analysis at each $m_{\pi\pi}$ with $\pi^+\pi^-$ amplitudes assumed to be nucleon-spin coherent and using absorption corrections (subsect. 4.7.3. (C)). (D) Analysis with a constant K-matrix fit using $\pi^+\pi^-$ and K^+K^-n data simultaneously (subsect. 4.7.4 (D)). (E) Analysis with $\pi^+\pi^-$ amplitudes assumed to be $\pi\pi$-spin coherent and using the ρ-meson line shape (subsect. 4.7.5 (E)).

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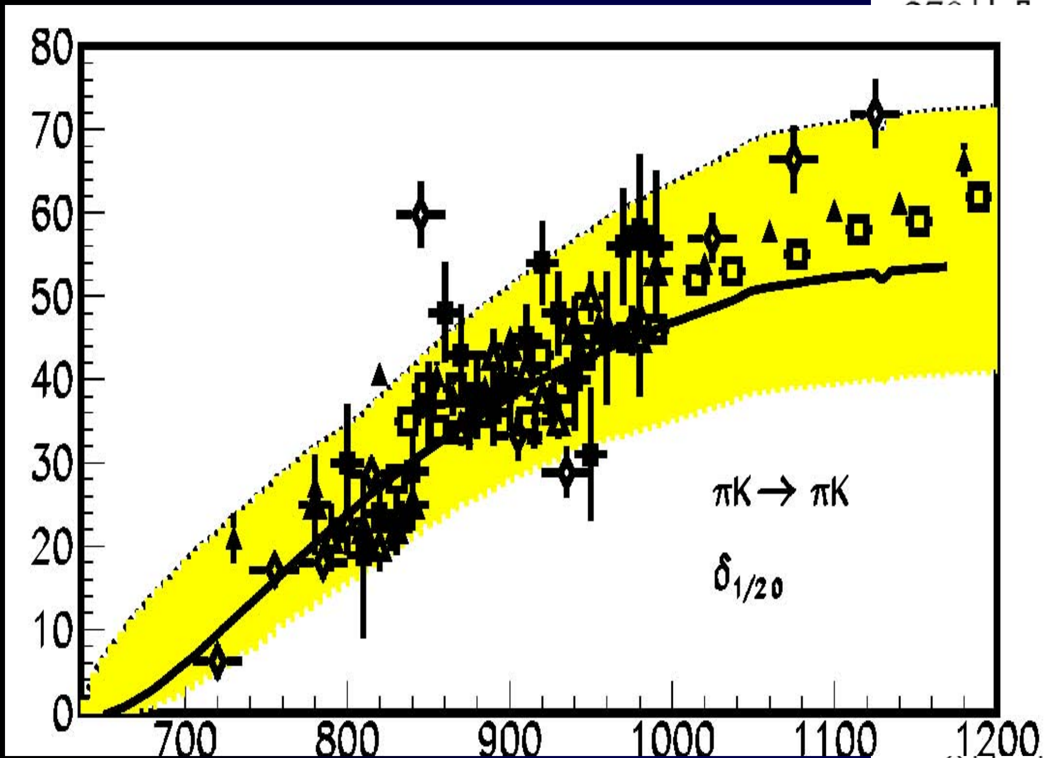
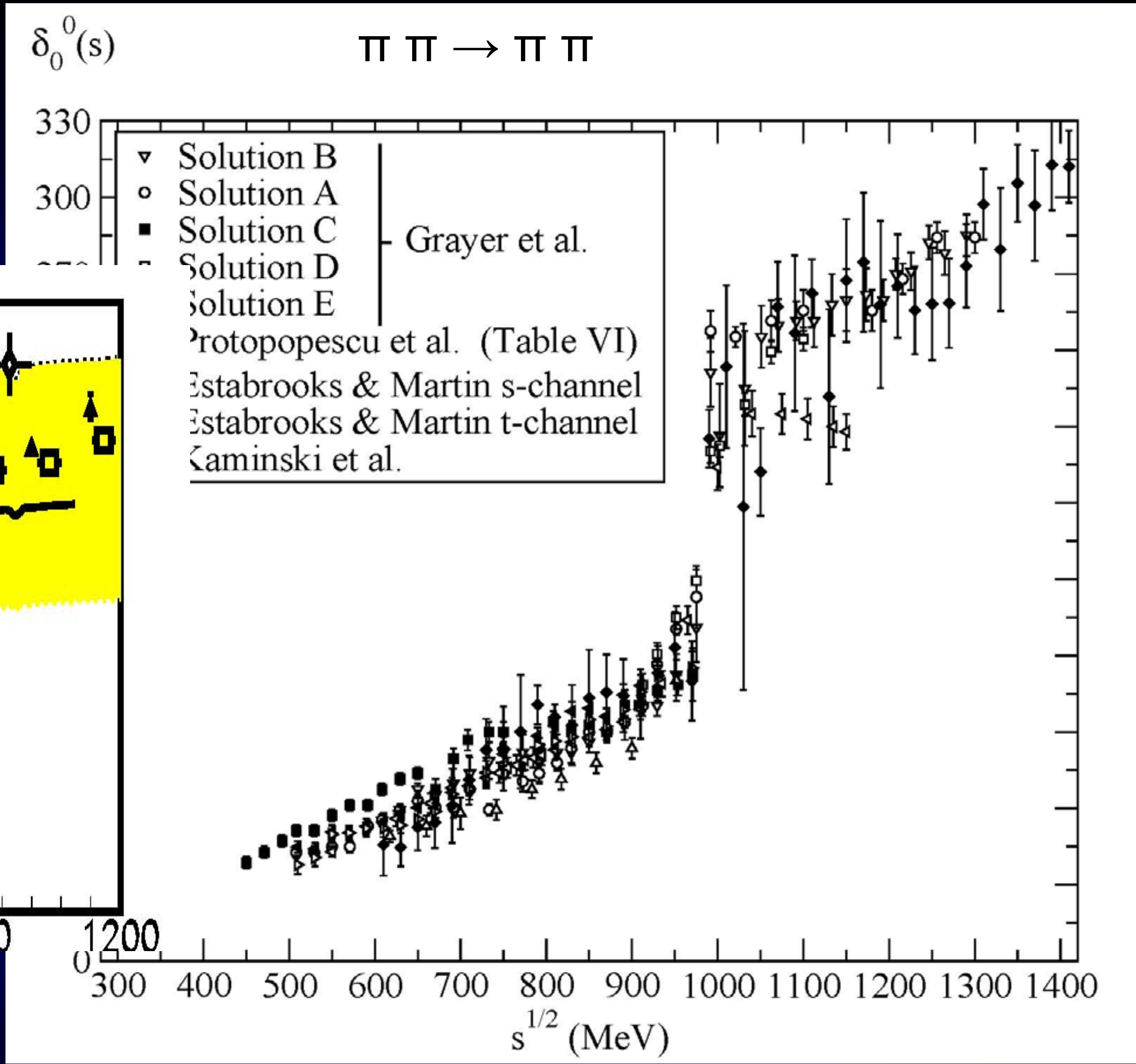
Motivation **ππ and πK SCATTERING data are often in conflict**

First problem:

CONFLICTING DATA SETS

From meson- Nucleon





This talk:

use DISPERSION RELATIONS to obtain data parameterizations:

Precise, consistent and EASY TO IMPLEMENT

ππ and πK SCATTERING data are bad

- π and K are unstable. Still, beams can be made.

But NOT luminous enough for **ππ and πK** collisions: **Indirect measurements**

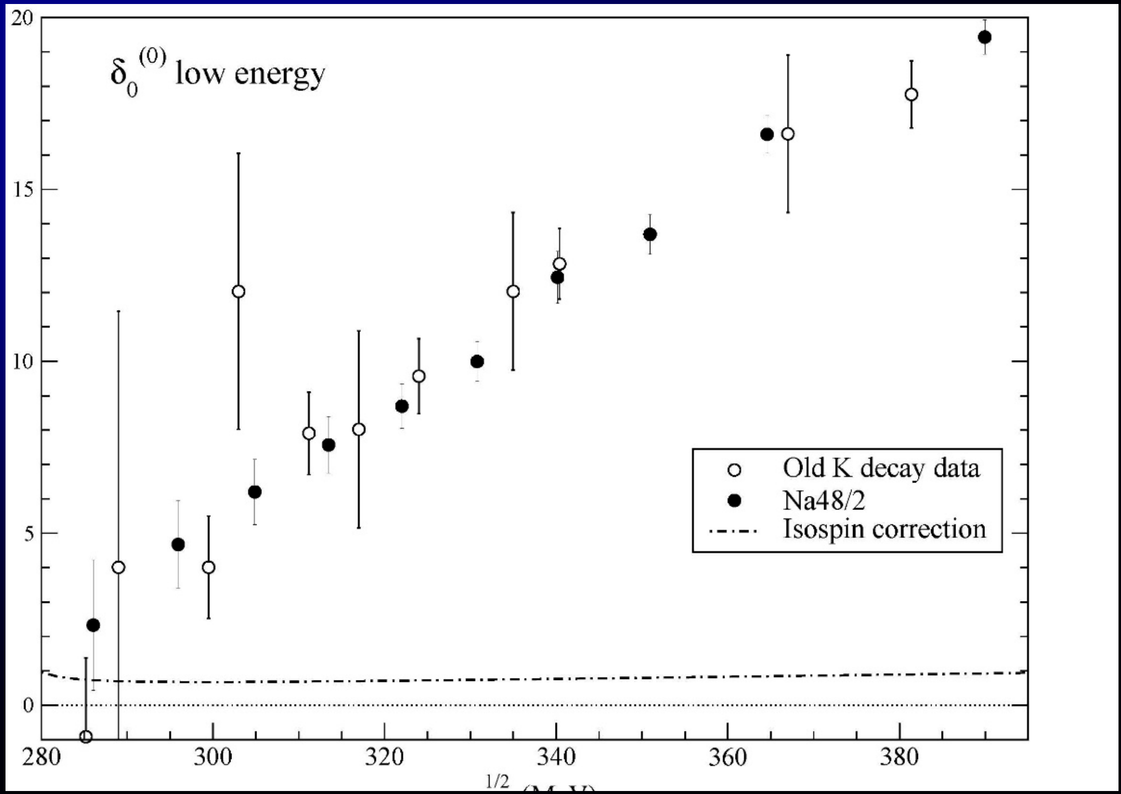
2) The only good data :From $K\to\pi\pi e\nu$ ("K_{l4} decays")

Geneva-Saclay (77), E865 (01), **NA48/2 (2010)**

Pions on-shell.

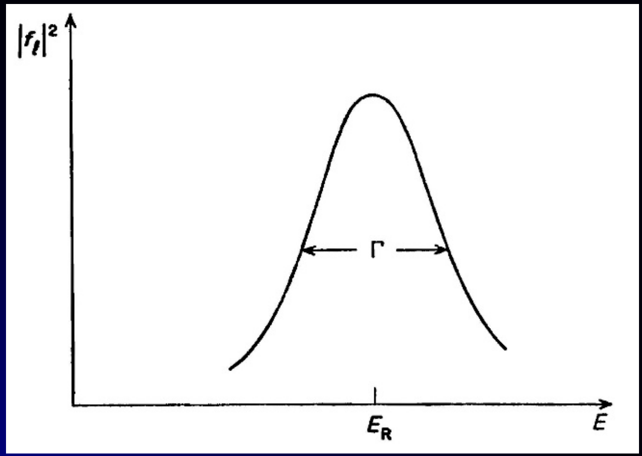
Very precise

BUT Limited:
only $\pi\pi\to\pi\pi$
only δ_{00}-δ_{11}.
only $E<M_K$

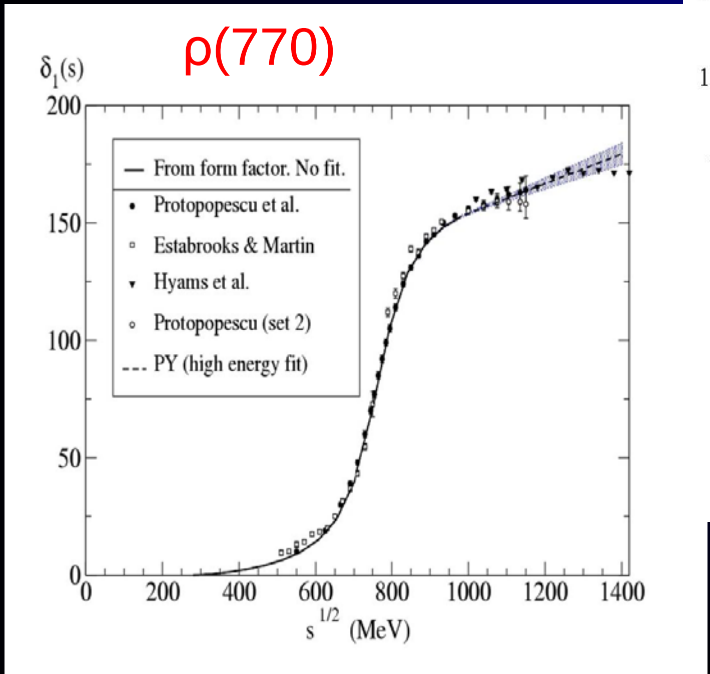


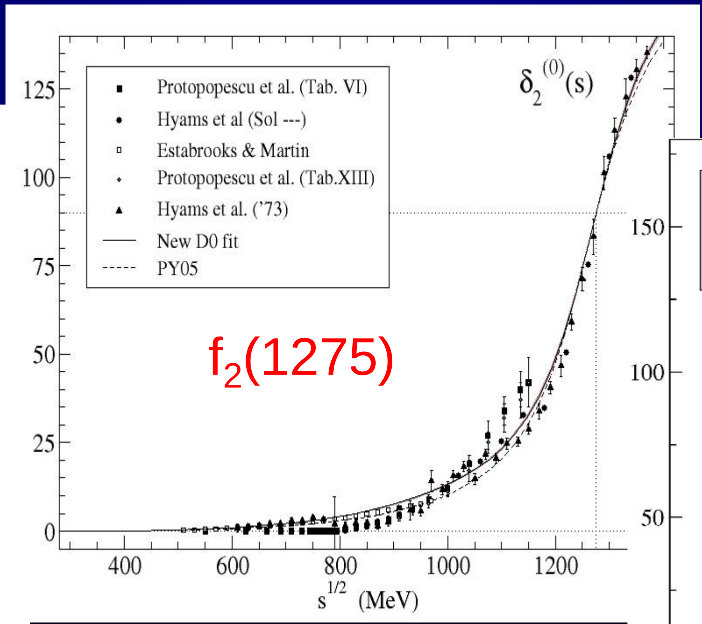
Usually, they are described by Breit-Wigner shapes

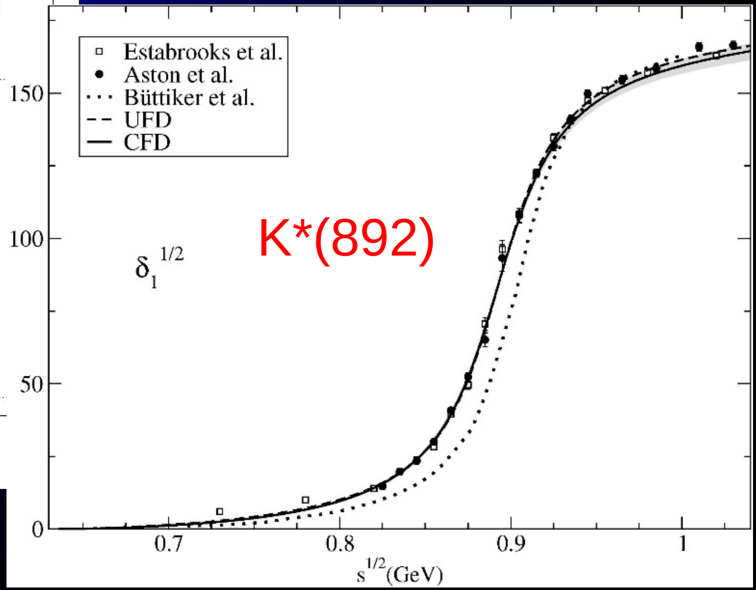
$$\sim \frac{M\,\Gamma(s)}{M^2 - s - iM\,\Gamma(s)}$$



Which in the elastic case produce a typical phase shift rapid increase from 0 to 180 degrees that we have already found several times



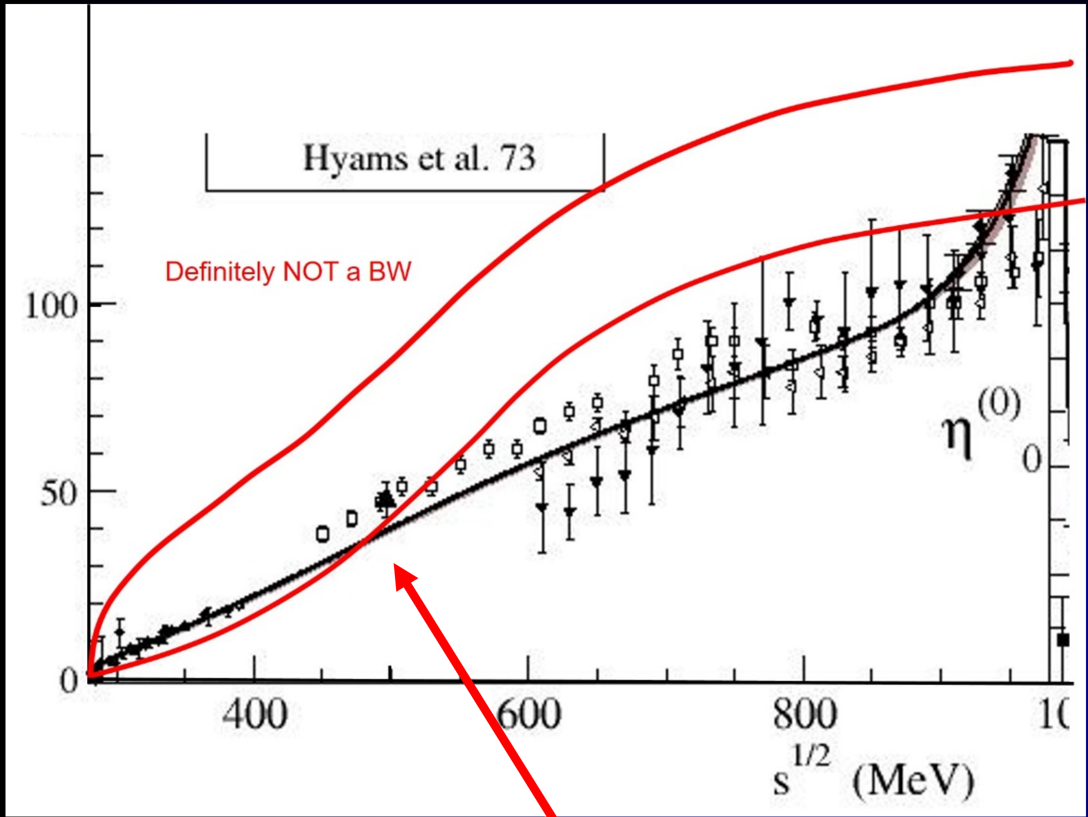


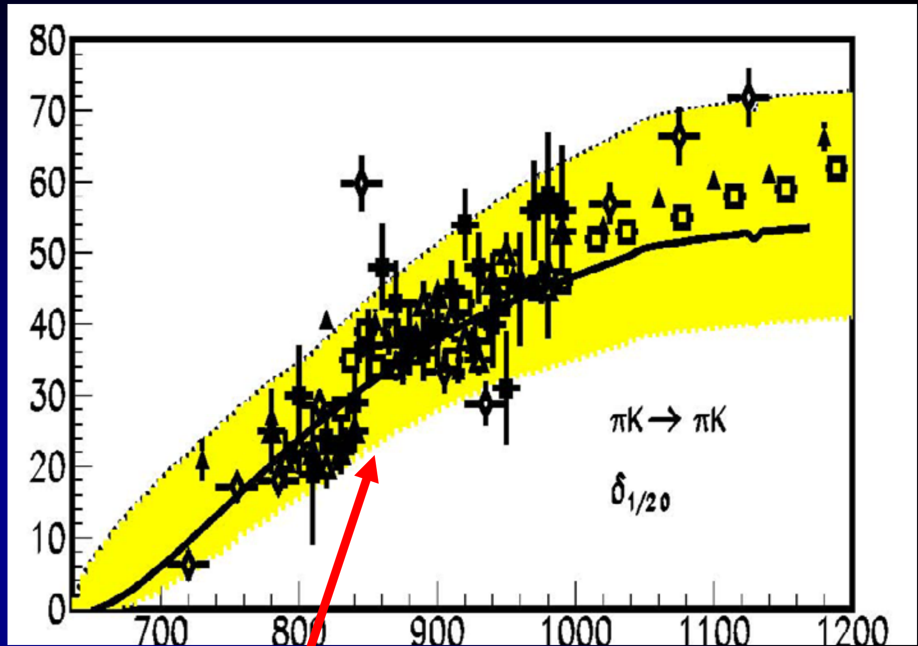


These are easily identified…

Motivation: Resonances in meson-meson scattering

Breit-Wigner shapes are easily recognizable…



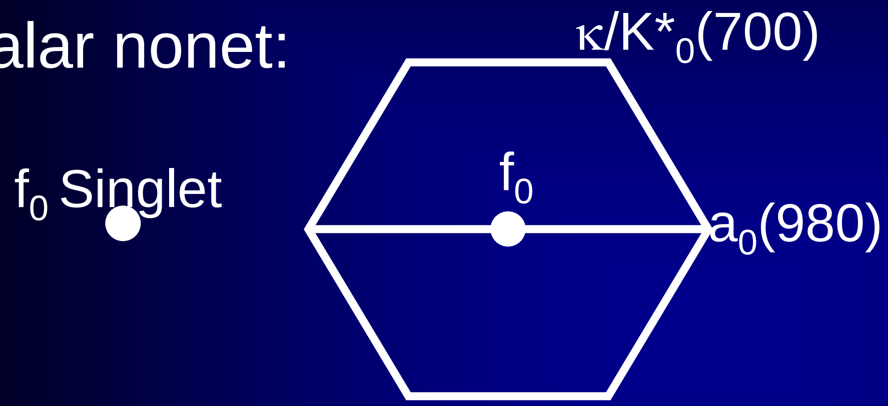


But do you see resonances there?

Nevertheless there is a resonance (a pole) on each graph: the $\sigma/f_0(500)$ and the $\kappa/K_0^*(800)$ light scalars

- Scalar SU(3) multiplets identification controversial
  - Too many resonances for many years.
    But there is an emerging picture…

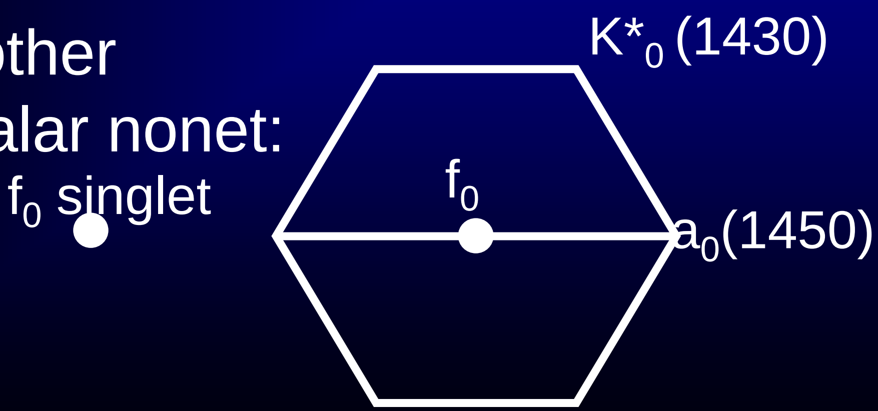
A Light scalar nonet:



Non-strange heavier!!
Inverted hierarchy problem
For quark-antiquark

f_0(500) and f_0(980) are
really OCTET/SINGLET mixtures

+ Another
heavier scalar nonet:



+ glueball

f_0

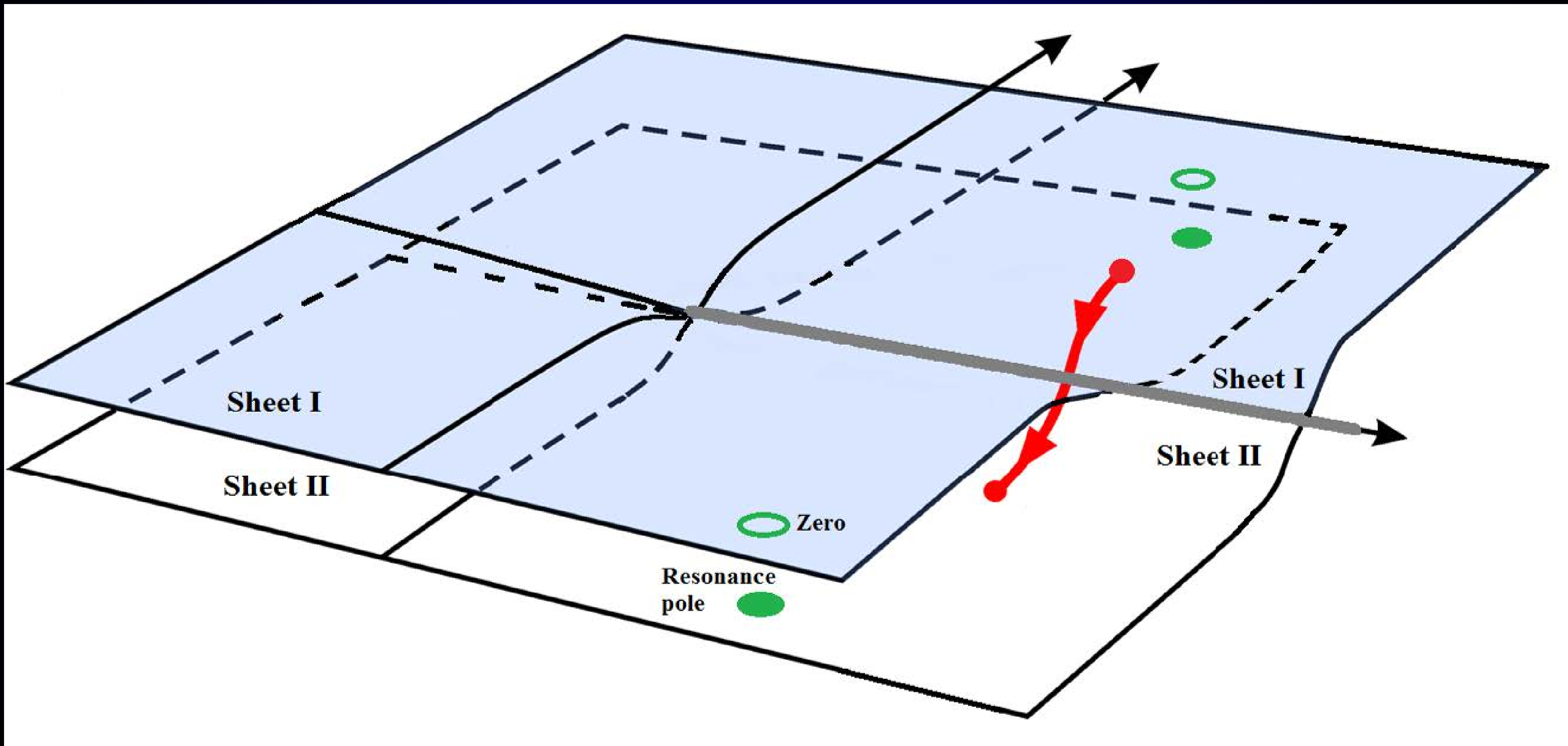
Enough f_0 states have been observed: f_0(500), f_0(980), f_0(1370), f_0(1500), f_0(1700).
The whole picture is complicated by mixture between them (lots of works here)

Resonances as poles

The Breit-Wigner shape is just an approximation for narrow and isolated resonances

The universal features of resonances are their
pole positions and residues *
$\sqrt{s_{pole}} \approx M - i\,\Gamma/2$

*in the Riemann sheet obtained from an analytic continuation through the physical cut

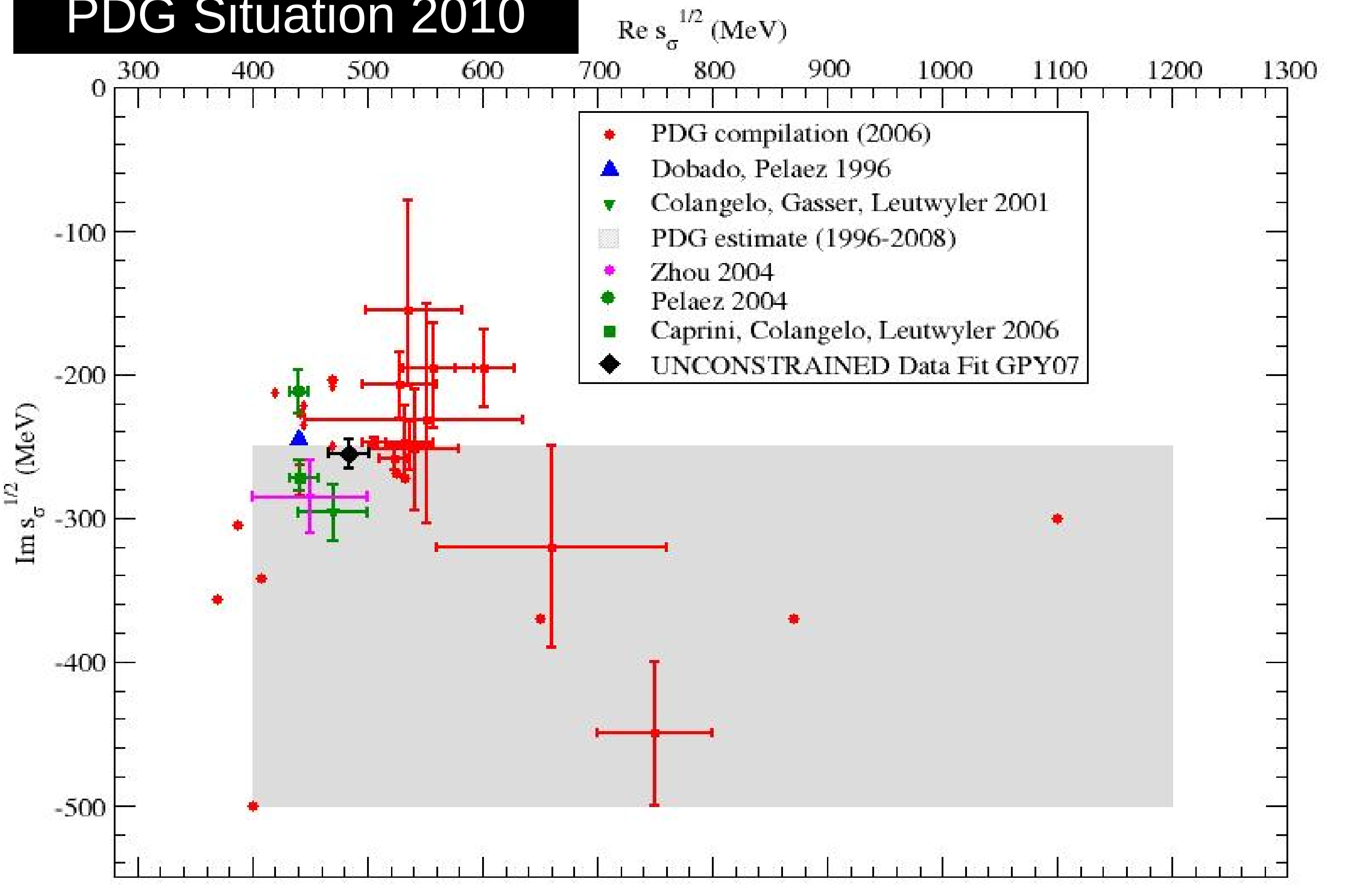


Very wide Resonance = pole deep in complex plane

Need correct analytic continuation

SIMPLE MODELS (like BW, or worse) created a mess

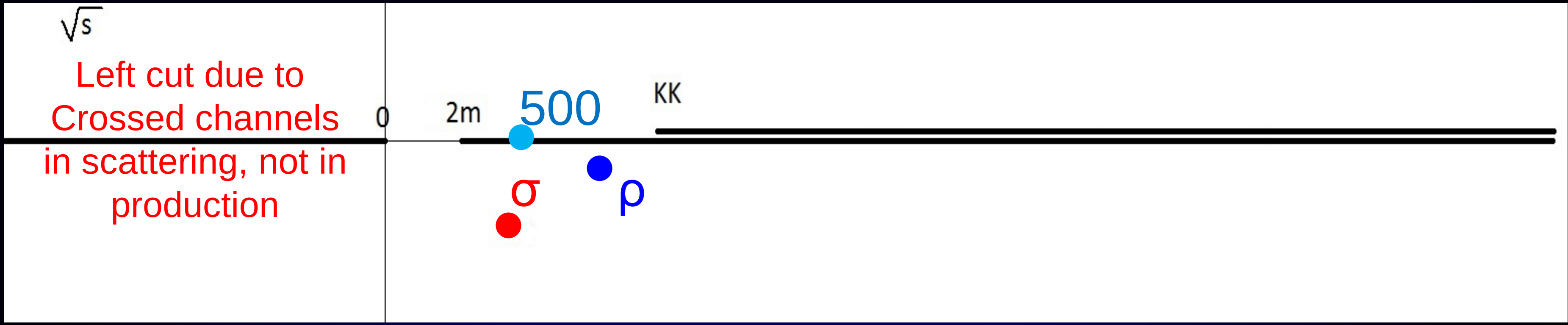
PDG Situation 2010



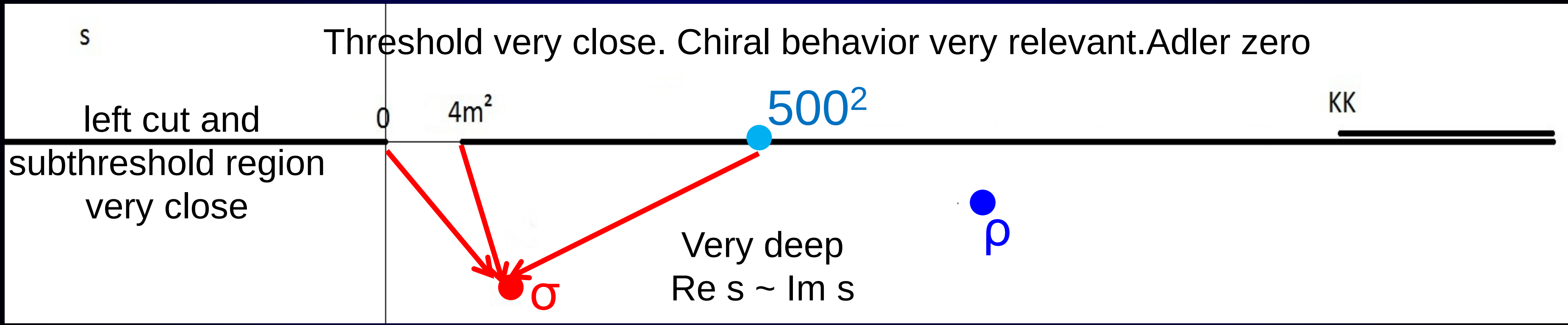
Need for dispersive formalism (analyticity) and chiral symmetry also relevant.

Why so much worries about low energy and CORRECT ANALYTIC STRUCTURE?

It is somewhat misleading to think of analyticity in terms of $\sqrt{s}$



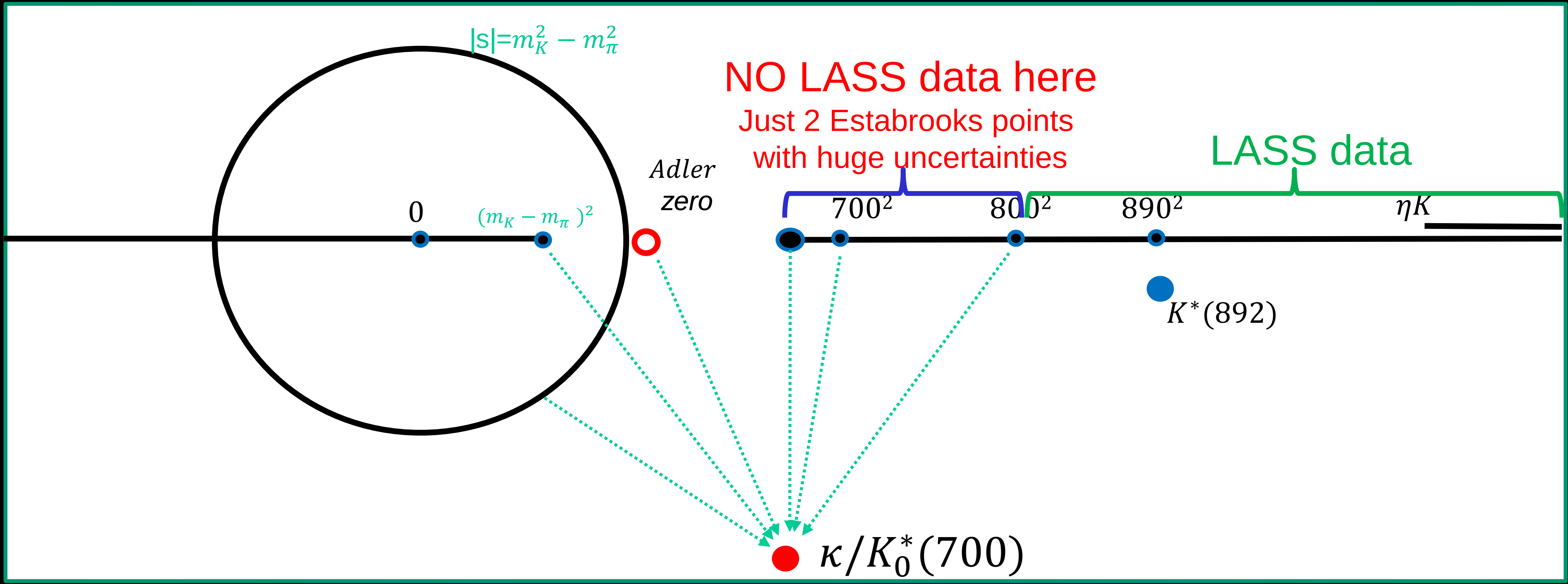
Since the partial wave is analytic in *s* ….



For the σ and κ a good control of the left cut and threshold region is important. This is why dispersion relations (Roy-like equations) are so relevant for precise pole determinations.

Why so much worries LOW ENERGY and CORRECT ANALYTIC STRUCTURE?

Analyticity is expressed in the *s*-variable, not in $\sqrt{s}$



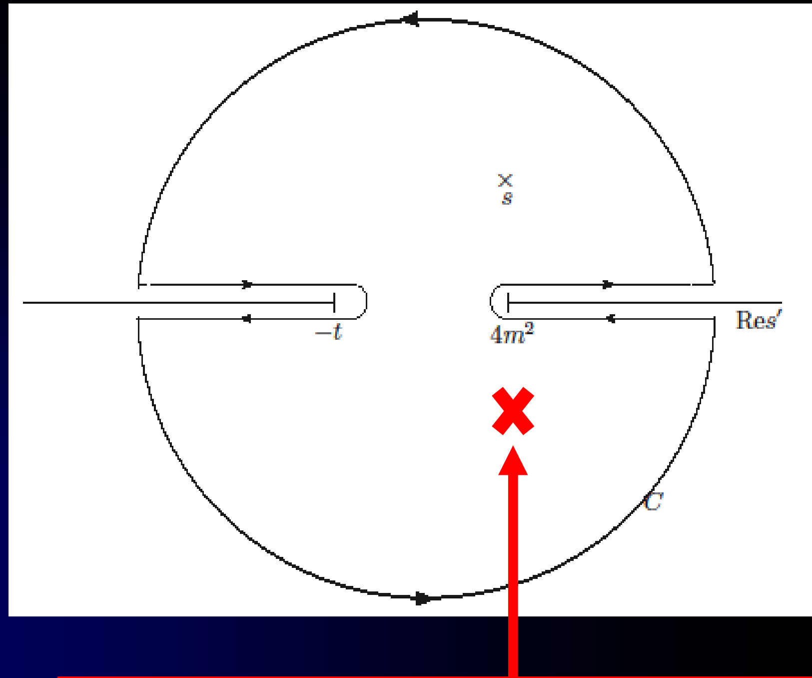
Important for the $\kappa/K_0^*(700)$

- Threshold behavior (Theory: chiral symmetry)
- Subthreshold behavior (Theory: chiral symmetry →Adler zeros)
- Other cuts (Theory: Left & circular)

Thus, LOW ENERGY behavior and ANALYTICITY crucial for the $K_0^*(700)$

What is a dispersion relation.? Very Briefly and for π π

CAUSALITY:
Partial waves t(s) are ANALYTIC in complex s plane
with cuts due to thresholds (also in crossed channels)



Cauchy Theorem determines t(s) at ANY s,
from an INTEGRAL on the contour

If t->0 fast enough at high s, curved part vanishes

$$t(s) = \frac{1}{\pi}\int_{th}^{\infty} \frac{Im\, t(s')}{s-s'} ds' + LC$$

Otherwise, deterr

We can calculate t(s)
anywhere we want using
the same integral expression

Good for:
1) Calculating t(s) where there is not data
2) Constraining data analysis
3) ONLY MODEL INDEPENDENT extrapolation to complex s-plane

So, we need to get rid of ONE VARIABLE
to write CAUCHY THEOREM in terms of the other one

1) Fix one variable in terms of the other
(fixed-t, hyperbolic relations…)

2) Integrate one variable and keep the other
(partial wave dispersión relations)

Single variable Dispersion Relations (DR)

- **1) Fixed-t Dispersion Relations** (or fixed-s) for $T(s,t_0)$

  Simple analytic structure in s-plane, simple derivation and use

  Left cut: With crossing may be rewritten in terms of physical region

  Most popular: $t_0=0$, **FORWARD DISPERSION RELATIONS** (FDRs).
  (Kaminski, Pelaez , Yndurain, Garcia Martin, Ruiz de Elvira, Rodas)

  One equation per amplitude.
  High Energy part very well known since Forward Amplitude~ Total cross section

  Positivity in the integrand contributions, good for precision.

  Calculated up to 1400 MeV (ππ) or 1.7 GeV (πK)

  Not practical for unphysical sheets

Forward dispersion relations for π π.

Complete isospin set of 3 forward dispersion relations for :

- Two s-u symmetric amplitudes. $F_{0+} \equiv \pi^0\pi^+ \to \pi^0\pi^+$, $F_{00} \equiv \pi^0\pi^0 \to \pi^0\pi^0$
  ONE SUBTRACTION
  Only depend on two isospin states. Positivity of imaginary part

$$\mathrm{Re}\,F(s) - \mathrm{Re}\,F(4M_\pi^2) = \frac{s(s-4M_\pi^2)}{\pi} PP \int_{4M_\pi^2}^{\infty} ds' \frac{(2s'-4M_\pi^2)\,\mathrm{Im}\,F(s')}{s'(s'-s)(s'-4M_\pi^2)(s'+s-4M_\pi^2)}$$

Additional sum rules SRJ, SRK if evaluated at $s=2M_\pi^2$ (Adler Zeros),

- The I_t=1 s-u antisymmetric amplitude

$$\mathrm{Re}\,F(s) = \frac{(2s-4M_\pi^2)}{\pi} PP \int_{4M_\pi^2}^{\infty} ds' \frac{\mathrm{Im}\,F(s')}{(s'-s)(s'+s-4M_\pi^2)}$$

At threshold is the Olsson sum rule

Partial-wave Dispersion Relations

Analytic structure complicated if unequal masses (Circular cuts)

Left cut: With crossing may be rewritten in terms of physical region.

But then different partial waves coupled.

In practice, limited to a finite energy.

But **good** and simple **for** elastic **resonance poles**

The second Riemann sheet in the elastic case

For elastic partial waves the second Riemann sheet is easy to obtain.

Due to elastic unitarity:

$$S^{II}(s) = \frac{1}{S^{I}(s)}$$

Recalling $\quad S(s) = 1 + 2i\sigma\, t(s), \quad \sigma(s) = \frac{k}{2\sqrt{s}}$

The second sheet is then:

$$t^{II}(s) = \frac{t^{I}(s)}{1 + 2i\sigma\, t^{I}(s)}$$

Looking for resonance poles
is nothing but looking for a zero in that denominator
on the first Riemann sheet accesible with the pw DR

The real improvement: Analyticity (and Effective Lagrangians)

- **Unitarized ChPT** 90's Truong, Dobado, Herrero, JRP, Oset, Oller, Ruiz Arriola, Nieves, Meissner,…

  Uses Chiral Perturbation Theory amplitudes inside dispersion relation.

  Relatively simple, although different levels of rigour. Generates all scalars

  Crossing (left cut) approximated… , not so good for precisión but good for connecting with QCD

- **Roy-like equations**. 70's Roy, Basdevant, Pennington, Petersen…

  00's Ananthanarayan, Caprini, Colangelo, Gasser, Leutwyler, Moussallam, Decotes Genon, Lesniak, Kaminski, JRP…

  Left cut implemented with precision . Use data on all waves + high energy .

  Optional: ChPT predictions for subtraction constants

  The most precise and model independent pole determinations

f_0(500) and K_0*(800) existence, mass and width

firmly established with precision

Roy-like Eqs. Derivation sketch

1) Choose the number of subtractions (2=Roy, 1=GKPY)

2) Write fixed-t dispersion relations and project them in partial waves.
Limited to s≤ 68 m_π^2 ~ O(1.1) GeV (More complicated extensions exist)

3) Use $s \leftrightarrow u$ crossing symmetry to re-write:

- left cut in terms of partial wave expansions of the other channels. But crossed channels are also ππ→ππ. Coupled equations.
- Subtraction terms

4) Truncate for low energy and low pw. The rest is input (driving terms)

Complications for πK→πK (Roy-Steiner Eqs). Also for πN and γγ→ππ)

2) Different masses. Better use "hyperbolic" Dispersion Relations for larger applicability domain.

3) Crossing involves other processes (ππ→KK). More equations coupled.

Both are coupled channel equations for the infinite partial waves:

I=isospin 0,1,2 , ℓ =angular momentum 0,1,2….

$$\mathrm{Re}\, t_\ell^{(I)}(s) = ST_\ell^{(I)}(s) + \sum_{I'=0}^{2}\sum_{\ell'=0}^{1} PP \int_{4M_\pi^2}^{s\max} ds'\, K_{\ell\ell'}^{II'}(s')\, \mathrm{Im}\, t_\ell^{(I)}(s') + DT_\ell^{(I)}(s)$$

Partial wave on real axis	SUBTRACTION TERMS (polynomials)	KERNEL TERMS known	DRIVING TERMS (truncation) Higher waves and High energy
	ROY: 2nd order	More energy suppressed	Very small
	GKPY: 1st order	Less energy suppressed	small

“OUT” =? “IN (from our data parametrizations)”

Two strategies

- SOLVE equations: (Ananthanarayan, Colangelo, Gasser, Leutwyler, Caprini, Moussallam, Stern…)

  S and P wave solution for Roy or GKPY equations unique at low energy if high-energy, higher waves and scattering lengths known. (in isospin limit)

  NO scattering DATA used at low energies ($\sqrt{s} \leq 0.8 \sim 1\ GeV$)

  Good if interested in low energy scattering and do not trust data.

  Uses ChPT input for threshold parameters

- Impose Dispersion Relations on fits to data. (García-Martín, Kaminski,JRP, Ruiz de Elvira, Ynduráin)

  Use any functional form and fit to DATA imposing DR within uncertainties.

  Also needs input on other waves and high energy.

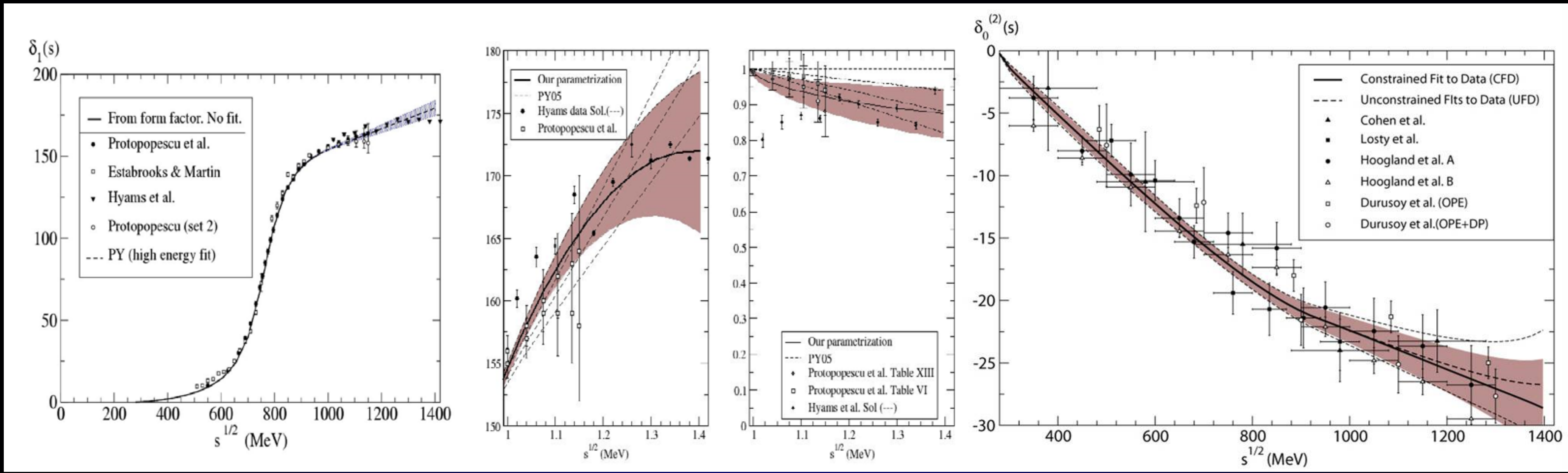
  (But you can use physical inspiration for clever choices of parameterizations)

Our series of works: 2005-2011

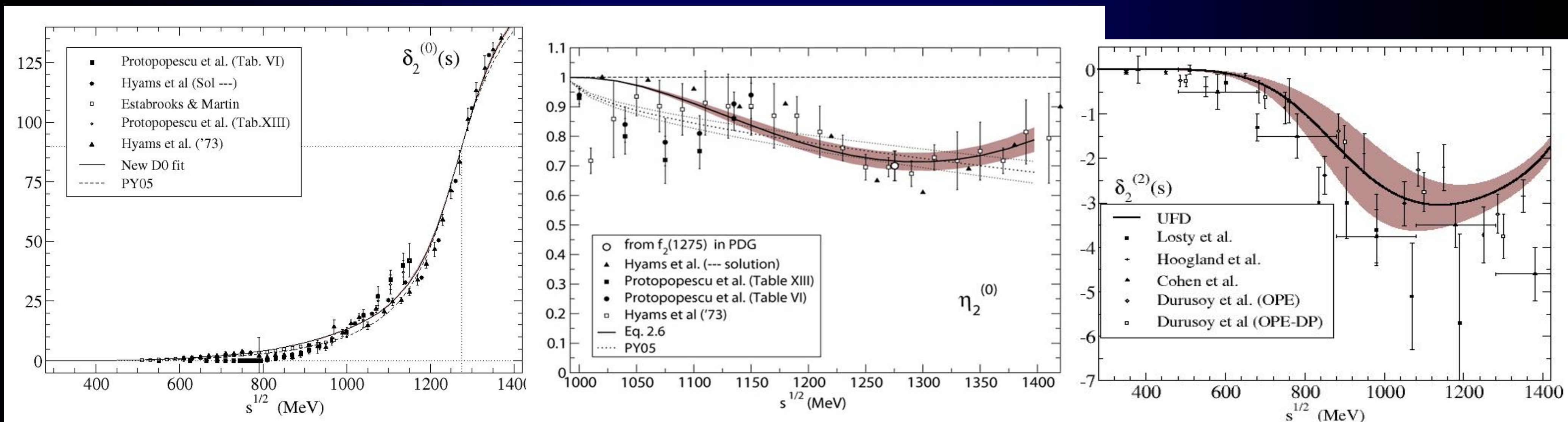
R. Kaminski, JRP, F.J. Ynduráin Eur.Phys.J.A31:479-484,2007. PRD74:014001,2006
JRP ,F.J. Ynduráin. PRD71, 074016 (2005) , PRD69,114001 (2004),
R. García Martín, R. Kaminski, JRP, J. Ruiz de Elvira, F.J. Ynduráin, Phys.Rev. D83 (2011) 074004,
R. García Martín, R. Kaminski, JRP, J. Ruiz de Elvira ,Phys.Rev.Lett. 107 (2011) 072001

Independent and **simple** fits
to data in different channels.
"**Unconstrained Data Fits=UDF**"

Simple UNconstrained Fits to Data: P wave, IJ=11

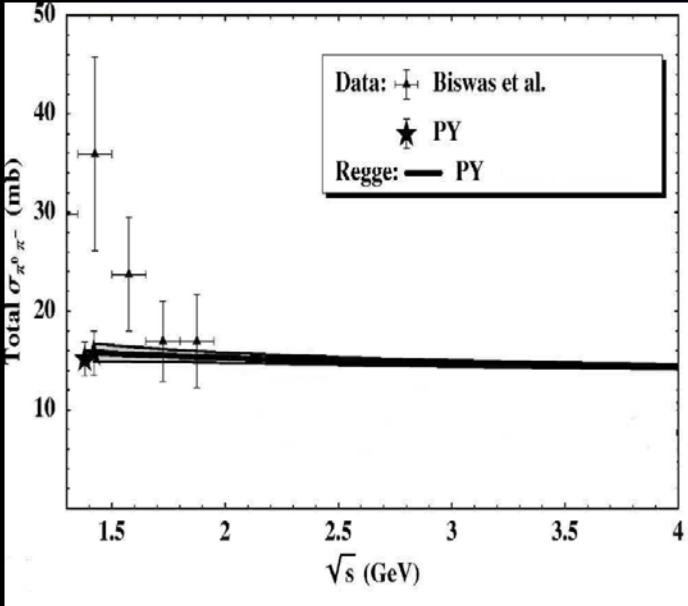


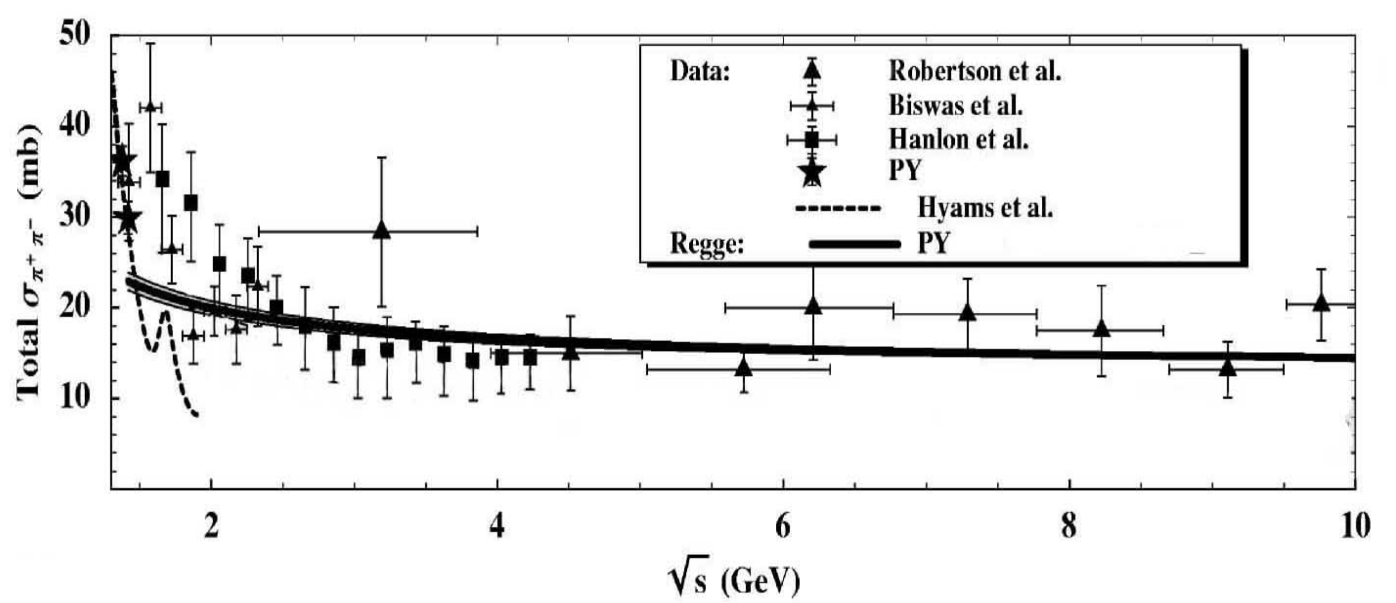
Simple fits easy to write down for phase shifts and inelasticities
For P,S2,D0,D2,anf F waves

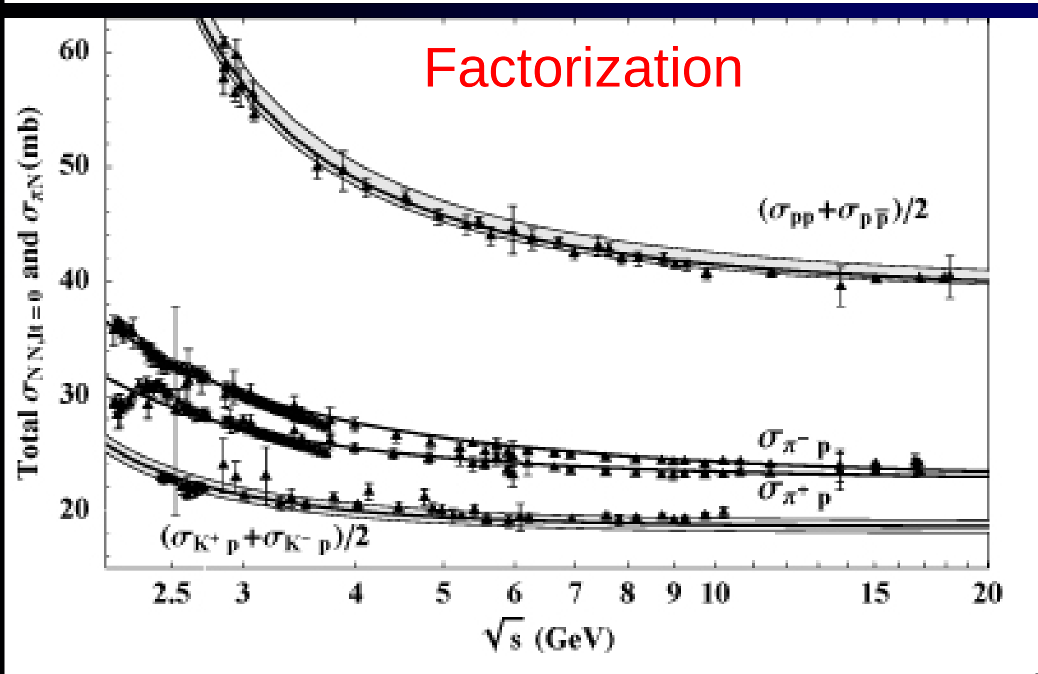


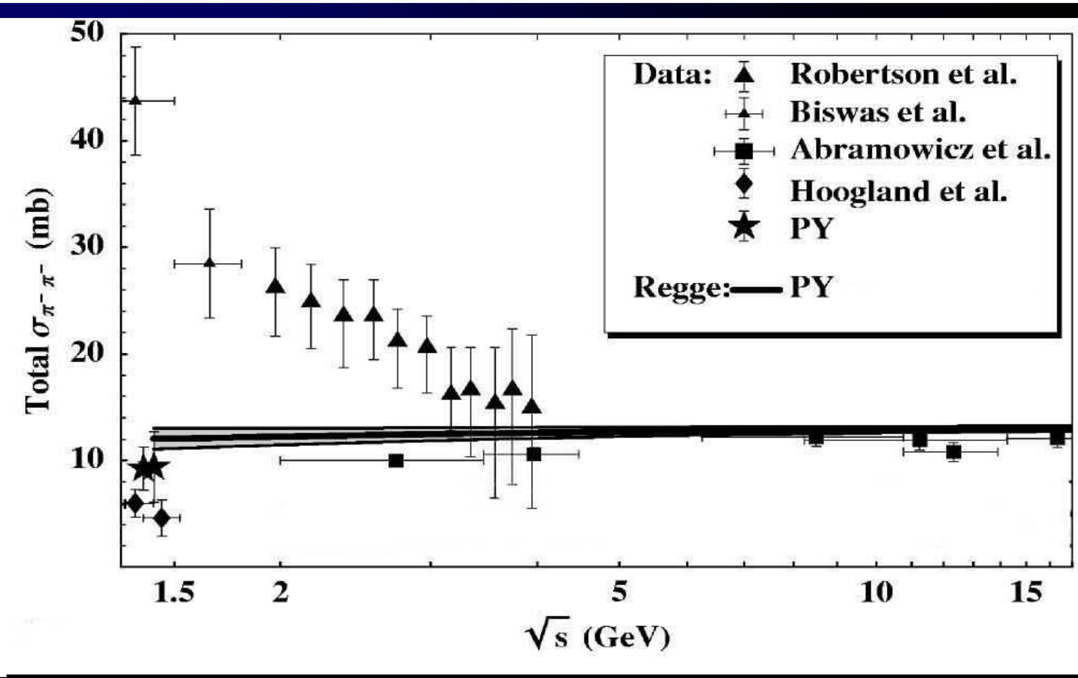
UNconstrained Fits for High energies

For simplicity we use **Regge parametrizations** of data JRP, F.J.Ynduráin. PRD69,114001 (2004)





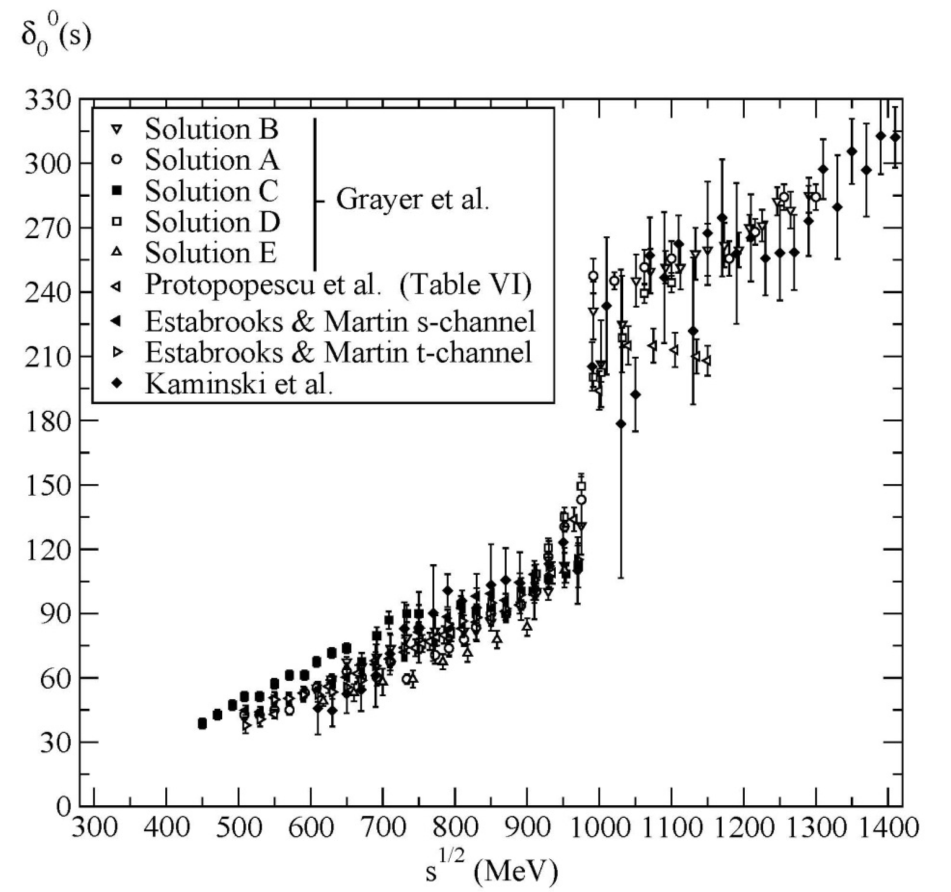


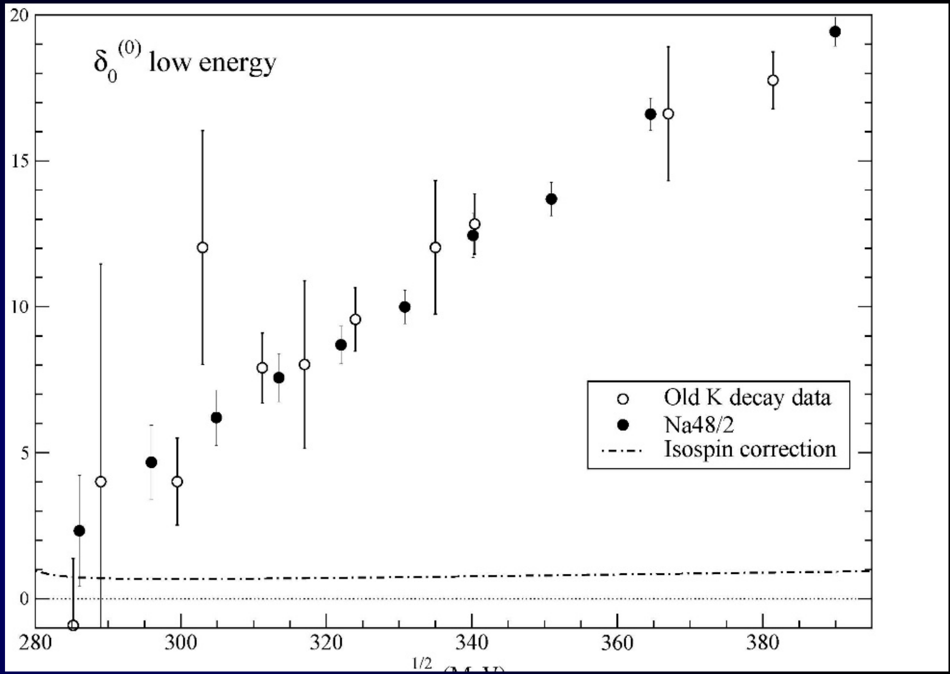


To be discussed later…

The complicated wave is the S0 wave (IJ=00)

We have already seen the data is a mess…. Only Kl4 reliable



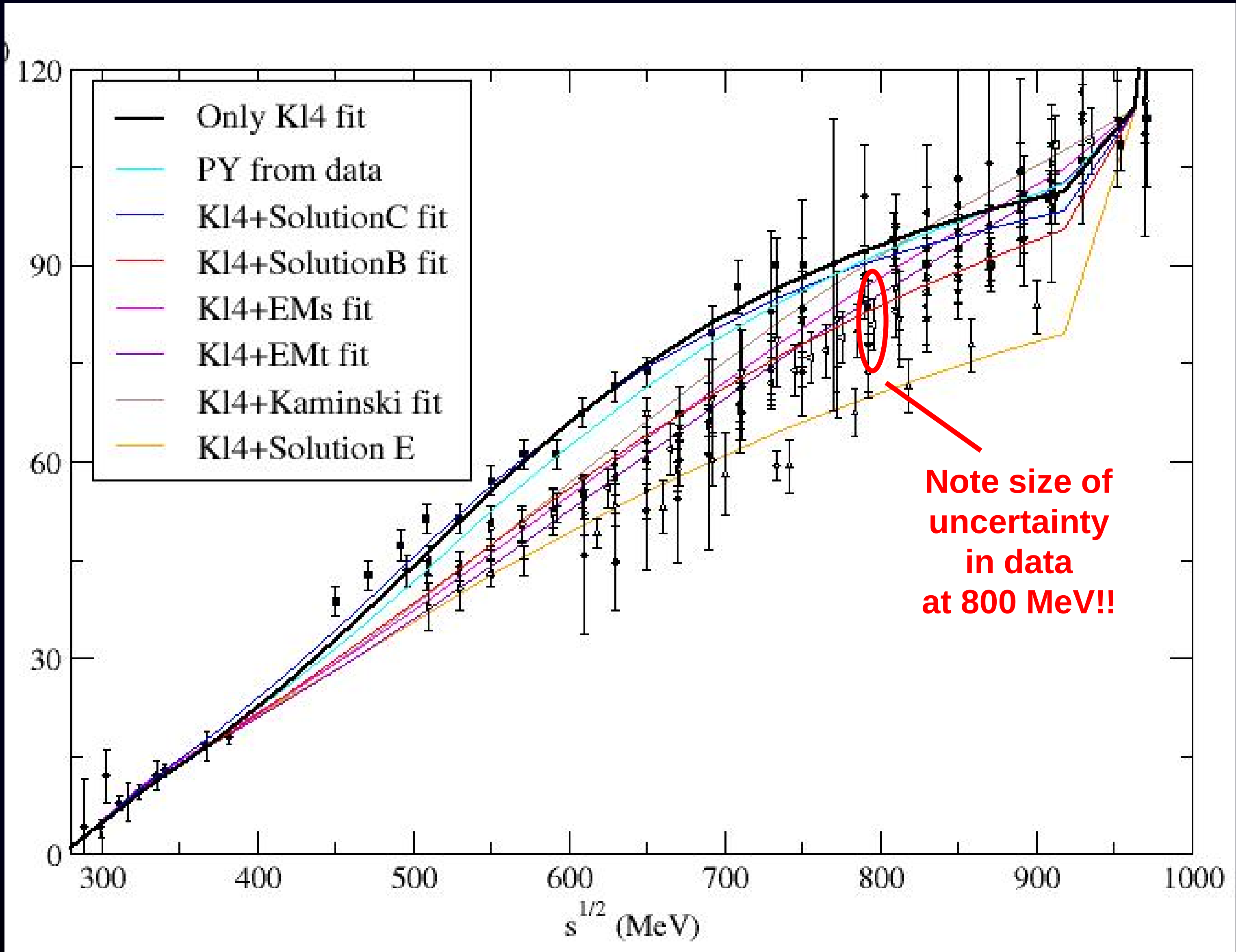


Always include Kl4, but two possibilities:

- Average data
- Fit individual sets

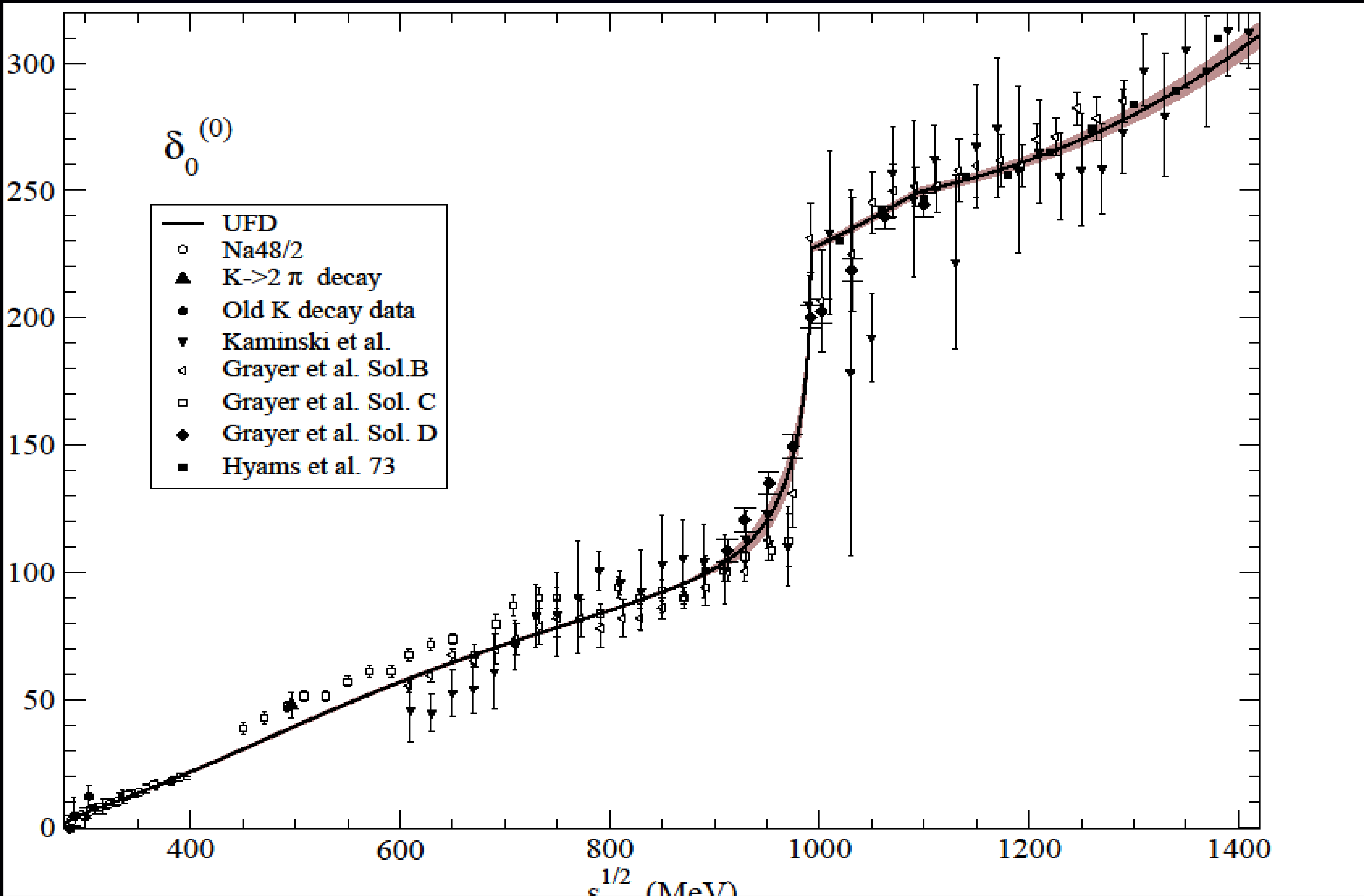
The S0 wave. Different sets

Fits to different sets including also K_{l4} data



S0 wave: UNconstrained fit to data (UFD).

Global fit, averaging all sets where they roughly coincide

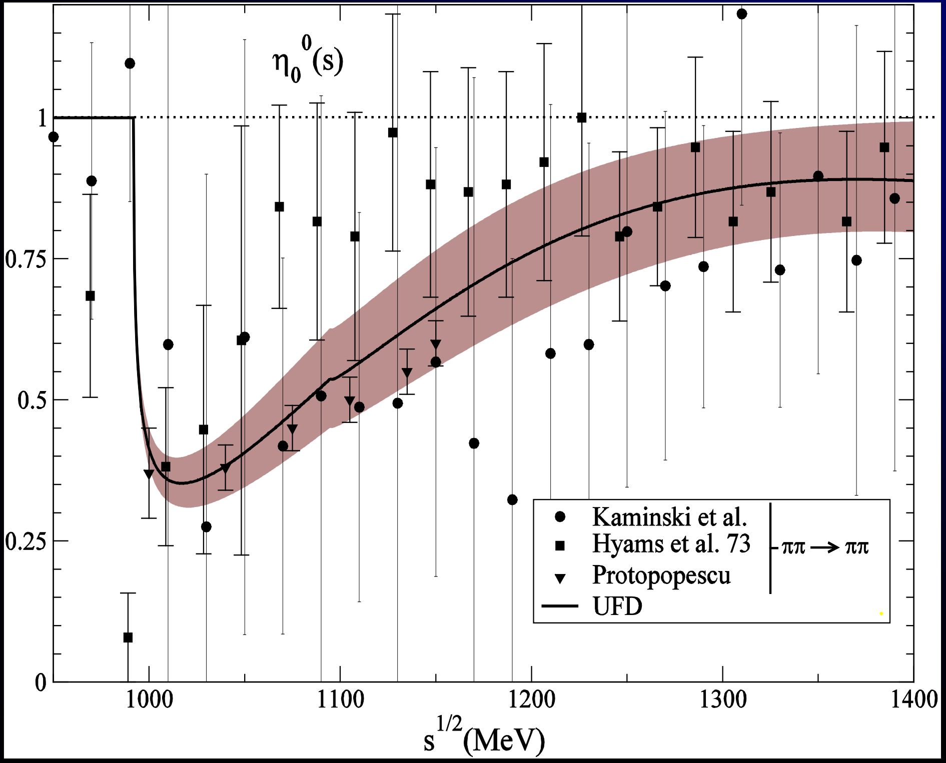


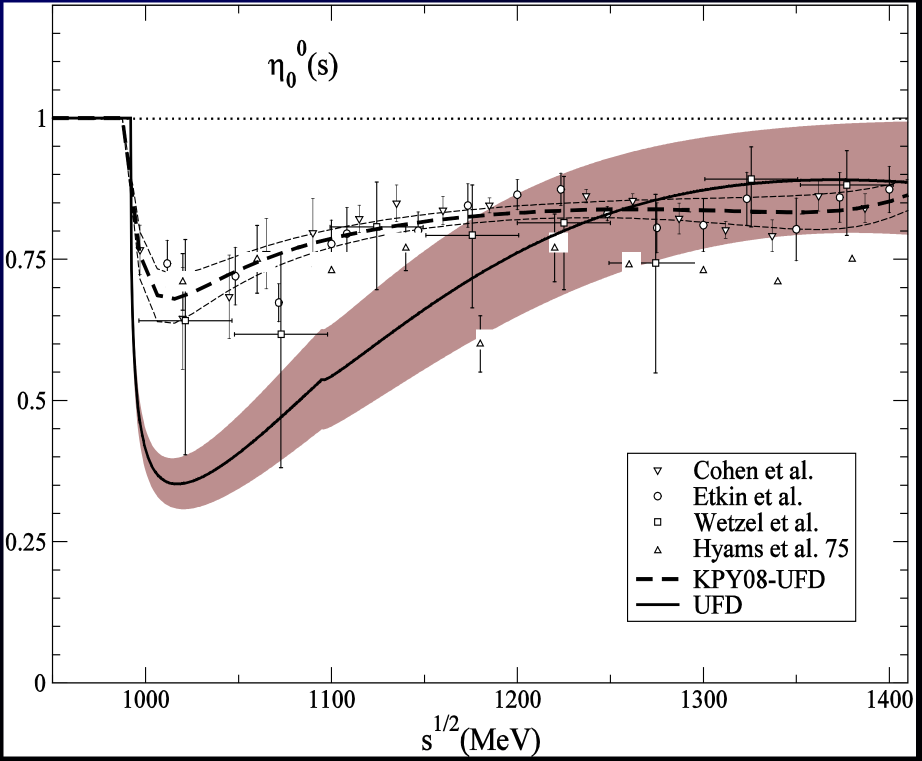
S0 wave: DIP vs NO DIP inelasticity scenarios

Longstanding controversy for inelasticity : (Pennington, Bugg, Zou, Achasov….)

There are inconsistent data sets for the inelasticity above 1 GeV near the $f_0(980)$ region

Some prefer a "dip" structure… ... whereas others do not





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Independent and **simple** fits
to data in different channels.
"**Unconstrained Data Fits=UDF**"

↓

Check Dispersion Relations

How well the Dispersion Relations are satisfied by unconstrained fits

Every 25 MeV we look at the difference between both sides of the DR divided by the uncertainty

We define an averaged χ^2 over these points, that we call d^2

d^2 close to 1 means that the relation is well satisfied

d^2>> 1 means the data set is inconsistent with the relation.

This is **NOT a fit** to the relation, just a check of the fits!!.

How well the Dispersion Relations are satisfied by unconstrained fits

Only TWO FDRs involve the S0 wave
The 00 FDR is very sensitive

Data sets	$\bar{d}^2$ for $T^{I_t=1}$	$\bar{d}^2$ for T^{00}
Global fit from [176]	0.3	3.5
K_{e4}+ Grayer et al. B	1.0	2.7
K_{e4}+ Grayer et al. C	0.4	1.0
K_{e4}+ Grayer et al. E	2.1	0.5
K_{e4}+ Kaminski et al.	0.3	5.0
K_{e4}+ Grayer et al. A	2.0	7.9
K_{e4}+ EM, s channel	1.0	9.1
K_{e4}+ EM, t channel	1.2	10.1
K_{e4}+ Protopopescu et al. VI	1.2	5.8
K_{e4}+ Protopopescu et al. XII	1.2	6.3
K_{e4}+ Protopopescu et al. VIII	1.8	4.2

Other sets, not so badly. Do not discad them but
ROOM FOR IMPROVEMENT

Some S0 data sets are very incompatible with FDR below 900 MeV
Considered clearly inconsistent and discarded

Lessons:

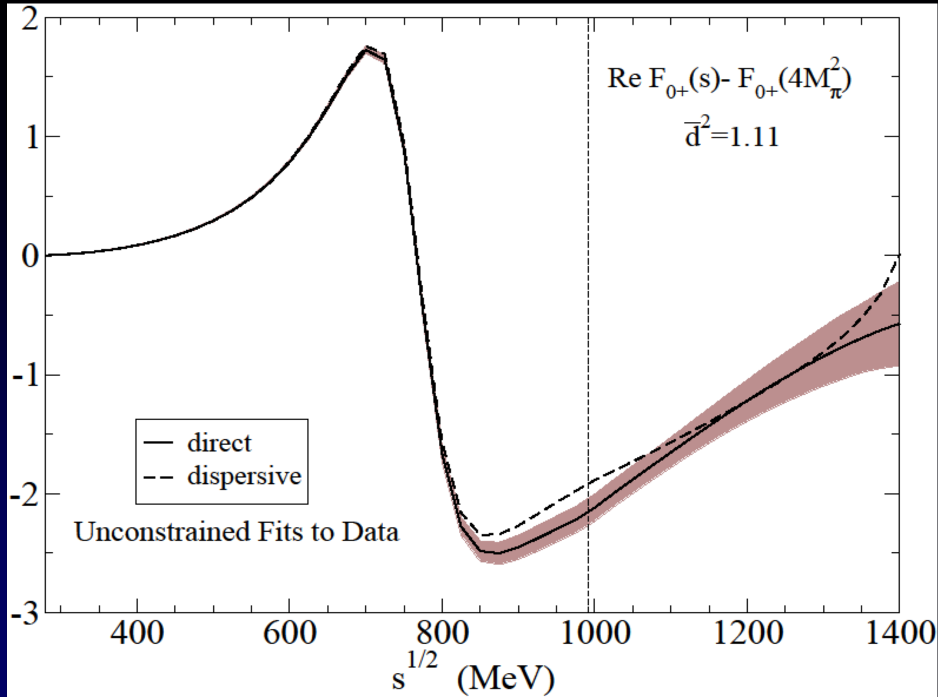
Dispersion Relations can be useful to discard conflicting data sets
Despite nice-looking fits, analytic properties WRONG.
Careful with extrapolations to complex plane

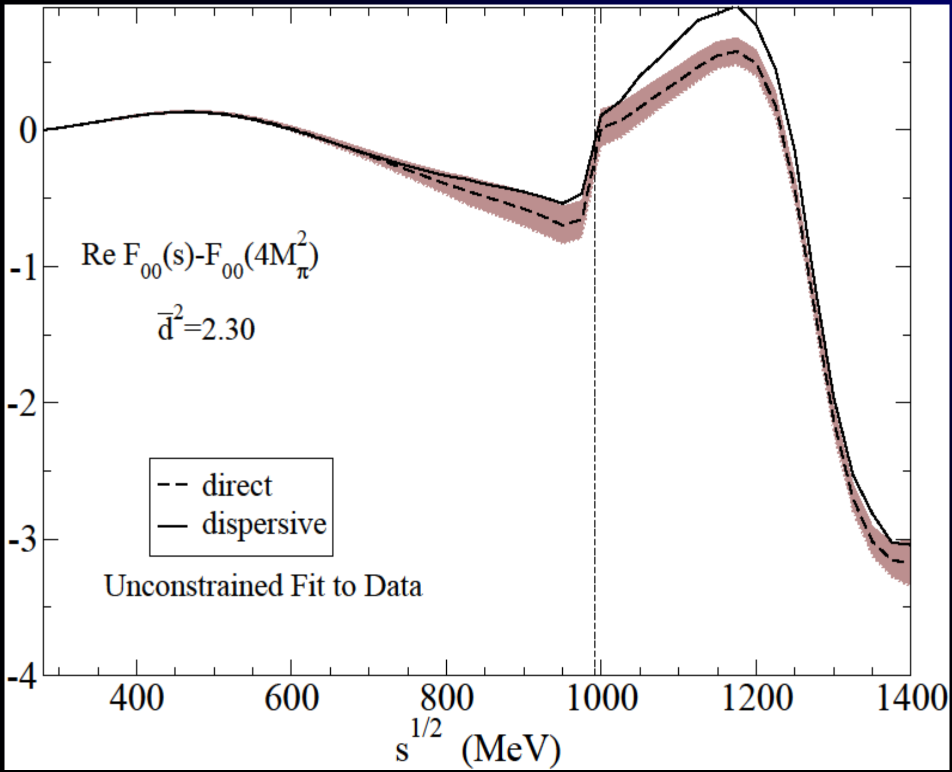
Forward Dispersion Relations for UNCONSTRAINED fits

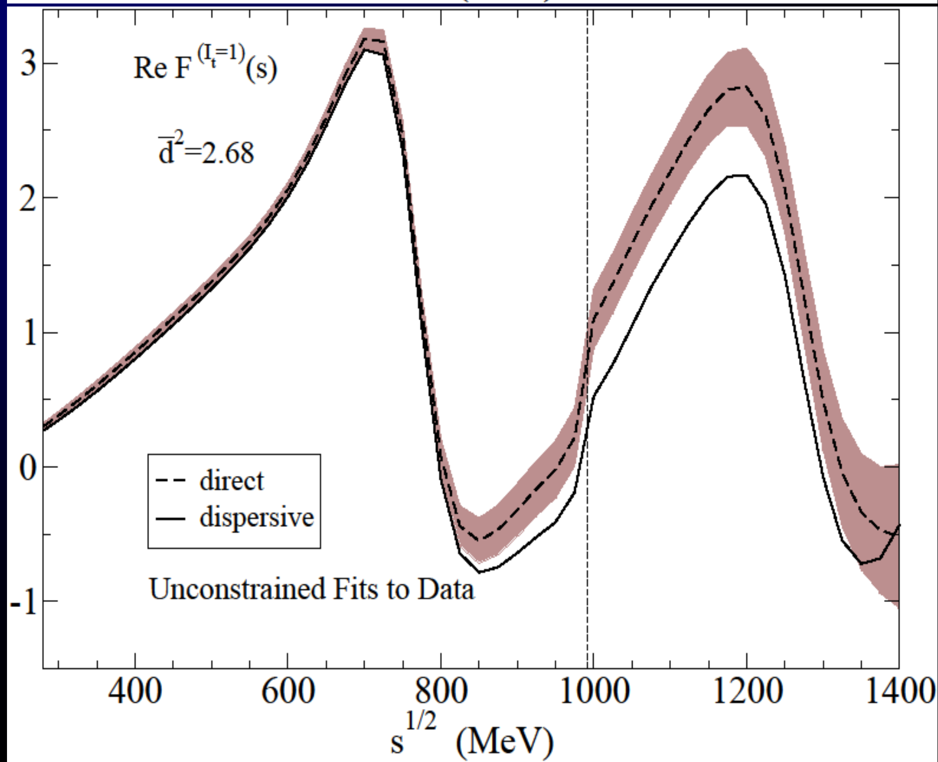
FDRs averaged d^2

	<932MeV	<1400MeV
$\pi^0\pi^0$	0.31	2.13
$\pi^0\pi^+$	1.03	1.11
I_t=1	1.62	2.69

NOT GOOD! In the intermediate region. Need improvement





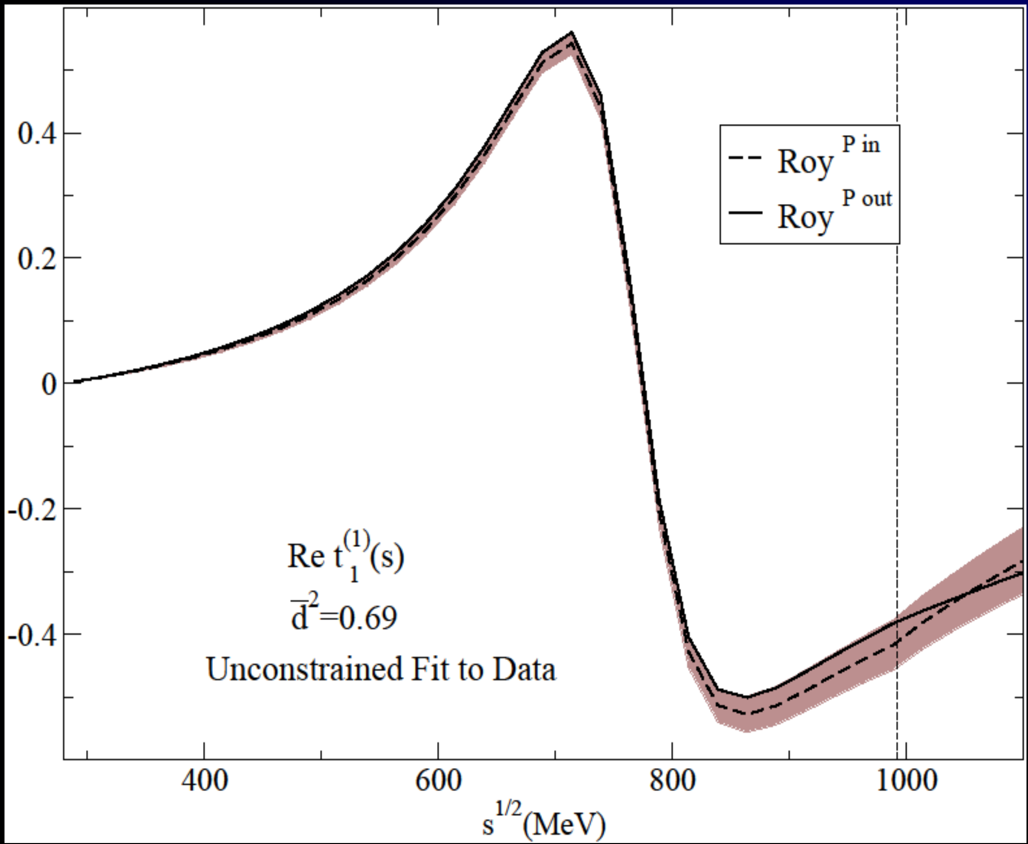


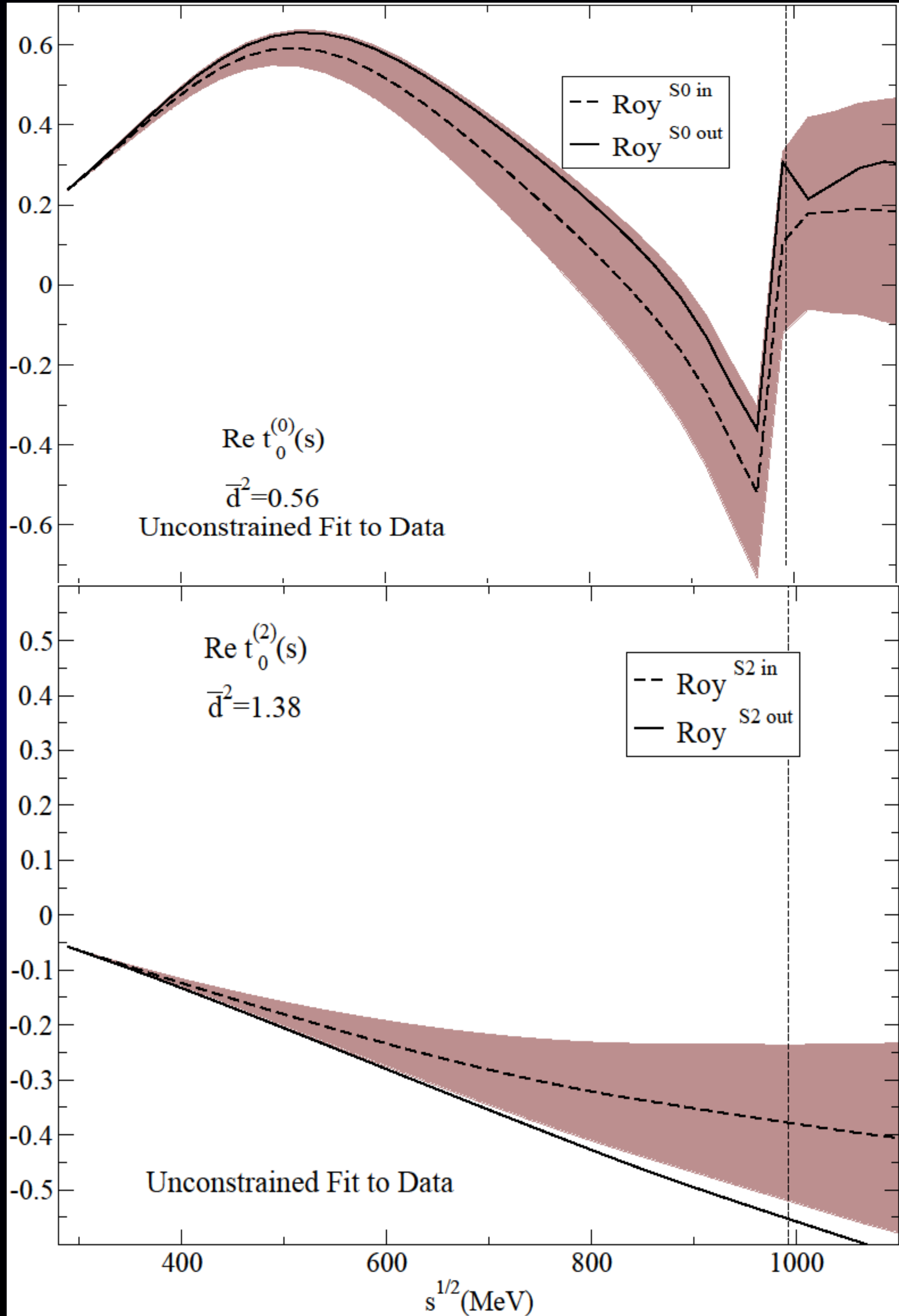
Roy Eqs. for UNCONSTRAINED fits

Roy Eqs. averaged d^2

	<932MeV	<1100MeV
S0wave	0.64	0.56
P wave	0.79	0.69
S2 wave	1.35	1.37

GOOD! But room for improvement





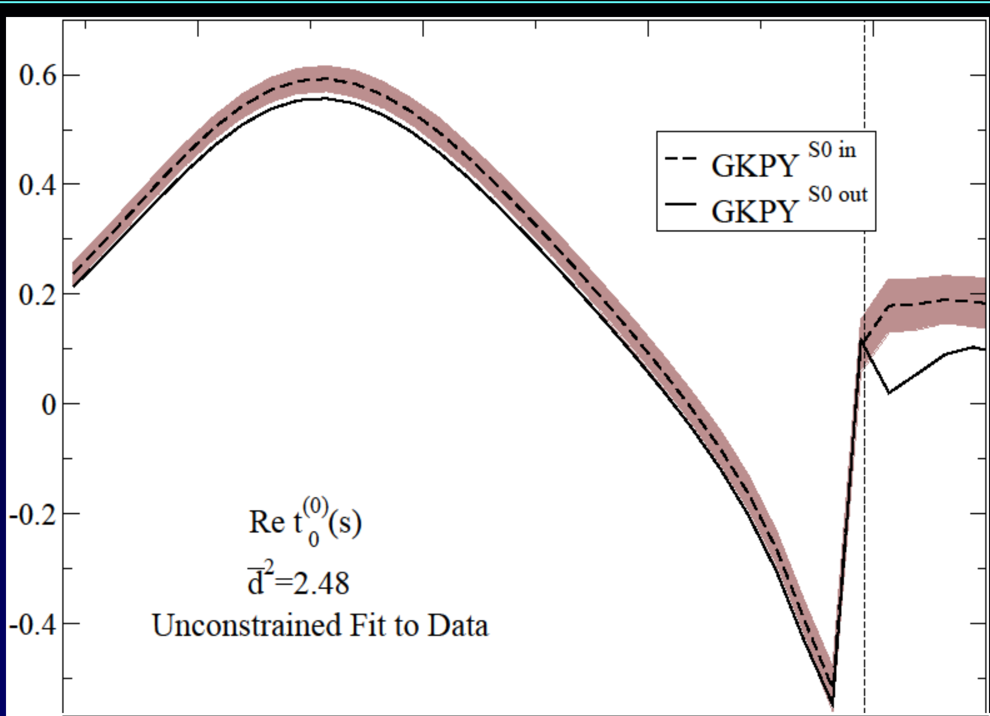
GKPY Eqs. for UNCONSTRAINED fits

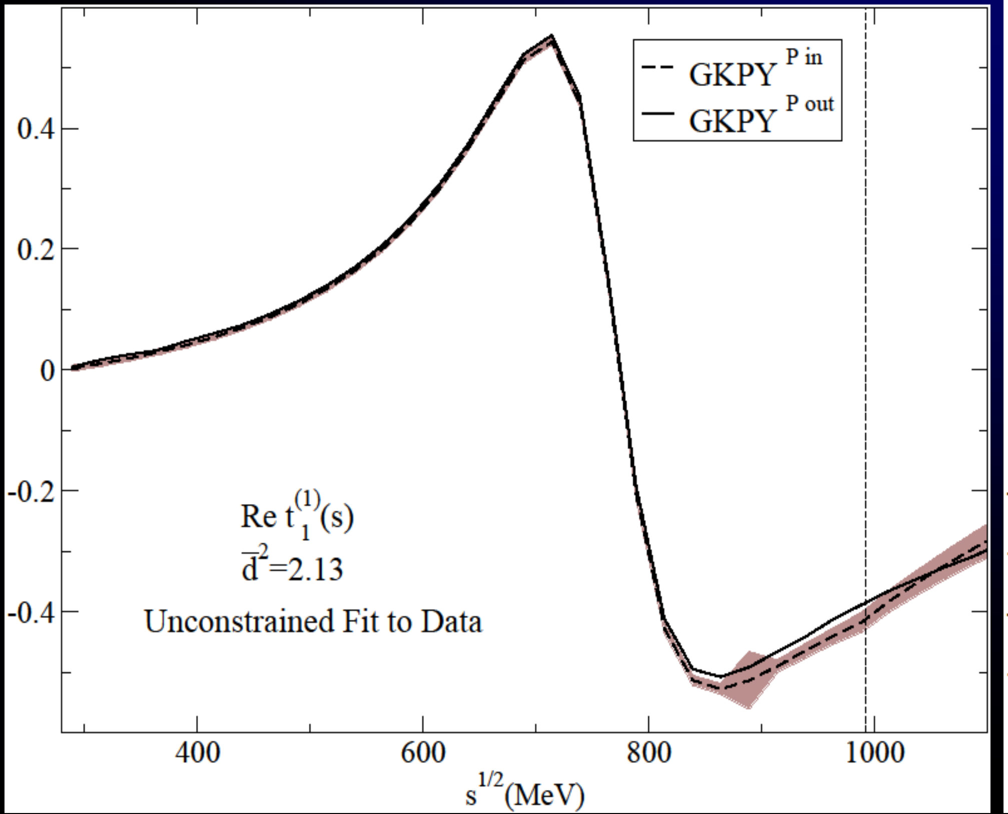
Roy Eqs. averaged d^2

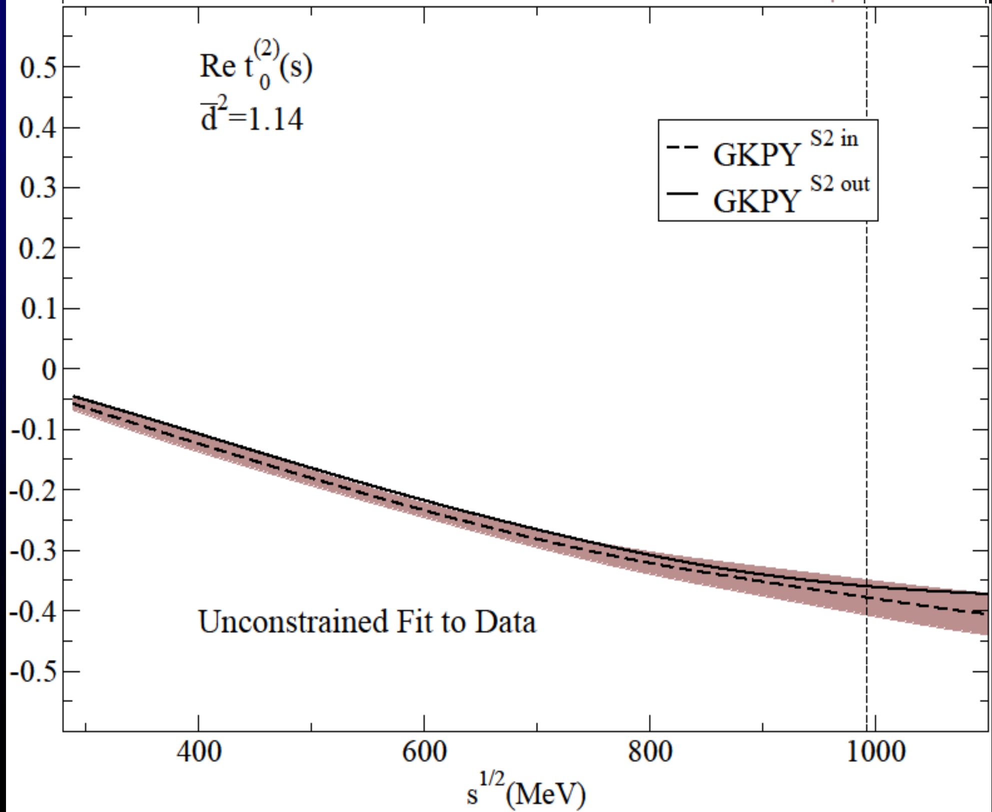
	<932MeV	<1100MeV
S0wave	1.78	2.42
P wave	2.44	2.13
S2 wave	1.19	1.14

Pretty bad. GKPY Eqs are much stricter

Lots of room for improvement







Our series of works: 2005-2011

R. Kaminski, JRP, F.J. Ynduráin Eur.Phys.J.A31:479-484,2007. PRD74:014001,2006
JRP ,F.J. Ynduráin. PRD71, 074016 (2005) , PRD69,114001 (2004),
R. García Martín, R. Kaminski, JRP, J. Ruiz de Elvira, F.J. Ynduráin, Phys.Rev. D83 (2011) 074004,
R. García Martín, R. Kaminski, JRP, J. Ruiz de Elvira ,Phys.Rev.Lett. 107 (2011) 072001

Independent and **simple** fits
to data in different channels.
“**Unconstrained Data Fits=UDF**”

↓

Check Dispersion Relations

Some sets discarded
Others, room for improvement

↓

Impose FDRs, Roy & GKPY Eqs
on data fits
“**Constrained Data Fits CDF**”
Describe data and are consistent with Dispersion relations

Imposing FDR's, Roy Eqs., GKPY Eqs. and crossing sum rules

To improve our fits, we can IMPOSE FDR's, Roy Eqs. GKPY Eqs. and some SRs

We obtain CONSTRAINED FITS TO DATA (CFD) by minimizing:

$$\chi^2 = W\left\{\overline{d_{00}}^2 + \overline{d_{0+}}^2 + \overline{d_{It=1}}^2 + \overline{d_{S0_{Roy}}}^2 + \overline{d_{S2_{Roy}}}^2 + \overline{d_{P_{Roy}}}^2 + \overline{d_{S0_{GKPY}}}^2 + \overline{d_{S2_{GKPY}}}^2 + \overline{d_{P_{GKPY}}}^2\right\}$$

3 FDR's — 3 Roy Eqs. — 3 GKPY Eqs.

$$+ \overline{d_{SR_J}}^2 + \overline{d_{SR_K}}^2 + \sum_k \frac{(p_k - p_k^{exp})^2}{\delta p_k^2}$$

Sum Rules for crossing

Parameters of the unconstrained data fits (could be data directly)

W roughly counts the number of effective degrees of freedom

(sometimes we add weight on certain energy regions)

Imposing FDRs and Sum rules

After imposing FDRs and SRs
The resulting fits differ by less than ~1σ -1.5 σ from original unconstrained fits

Data Fits Constrained with FDR	$\bar{d}^2$ for $T^{I_t=1}$	$\bar{d}^2$ for T^{00}	K
Global fit from [176]	0.4	0.66	1.6σ
K_{e4}+ Grayer et al. C	0.37	0.32	1.5σ
K_{e4}+ Grayer et al. B	0.37	0.83	4.0σ
K_{e4}+ Grayer et al. E	0.6	0.09	6.0σ
K_{e4}+ Kaminski et al.	0.43	1.08	4.5σ

Remarkable improvement in 00 FDR

But some sets cannot be made to satisfy SR: DISCARDED

Fit C included within uncertainties of “Global Fit”.

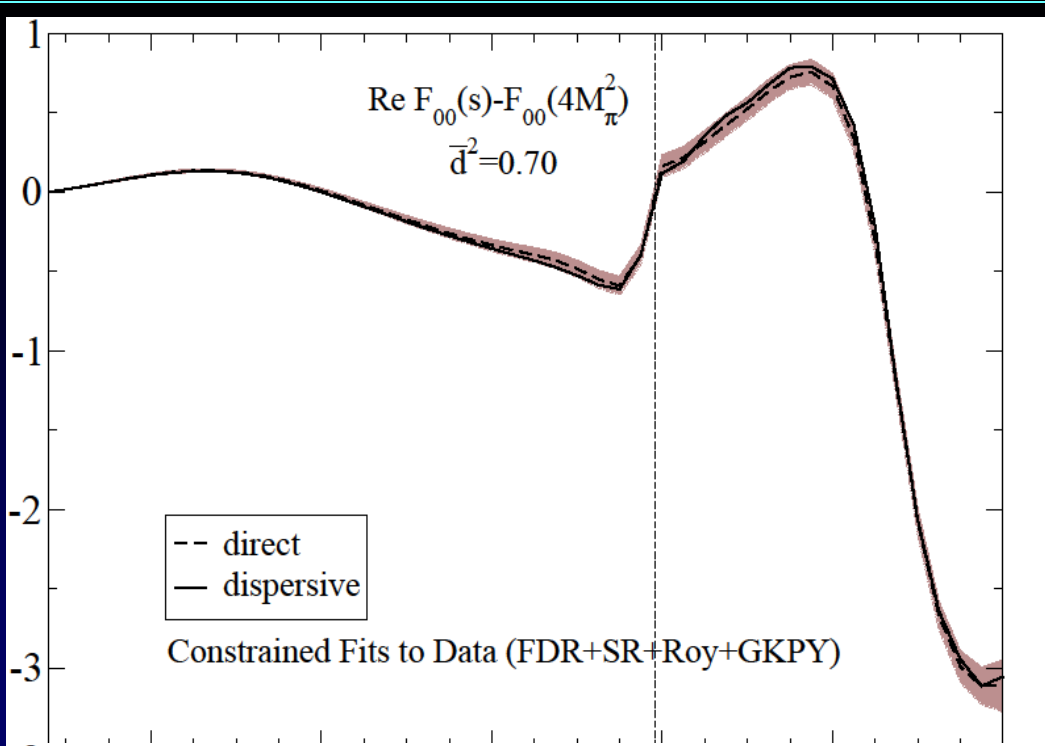
So **<u>we keep the “Global Fit”</u>**

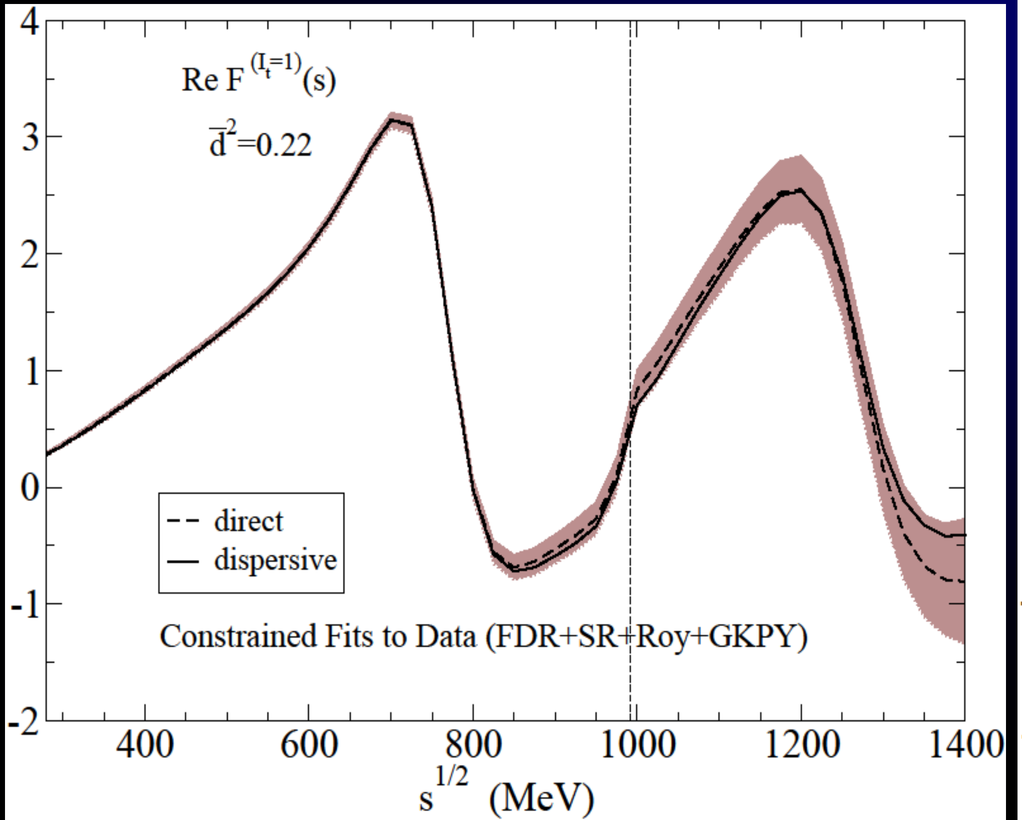
Forward Dispersion Relations for CONSTRAINED fits

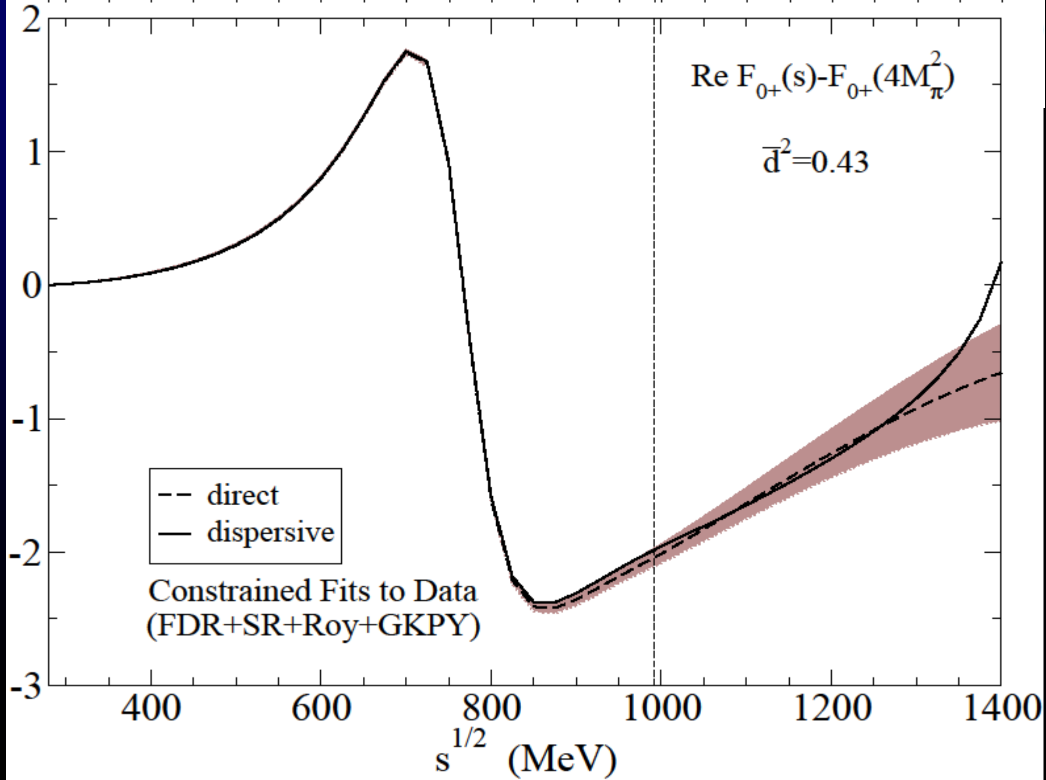
FDRs averaged d^2

	<932MeV	<1400MeV
$\pi^0\pi^0$	0.32	0.51
$\pi^0\pi^+$	0.33	0.43
$I_t=1$	0.06	0.25

VERY GOOD!!!





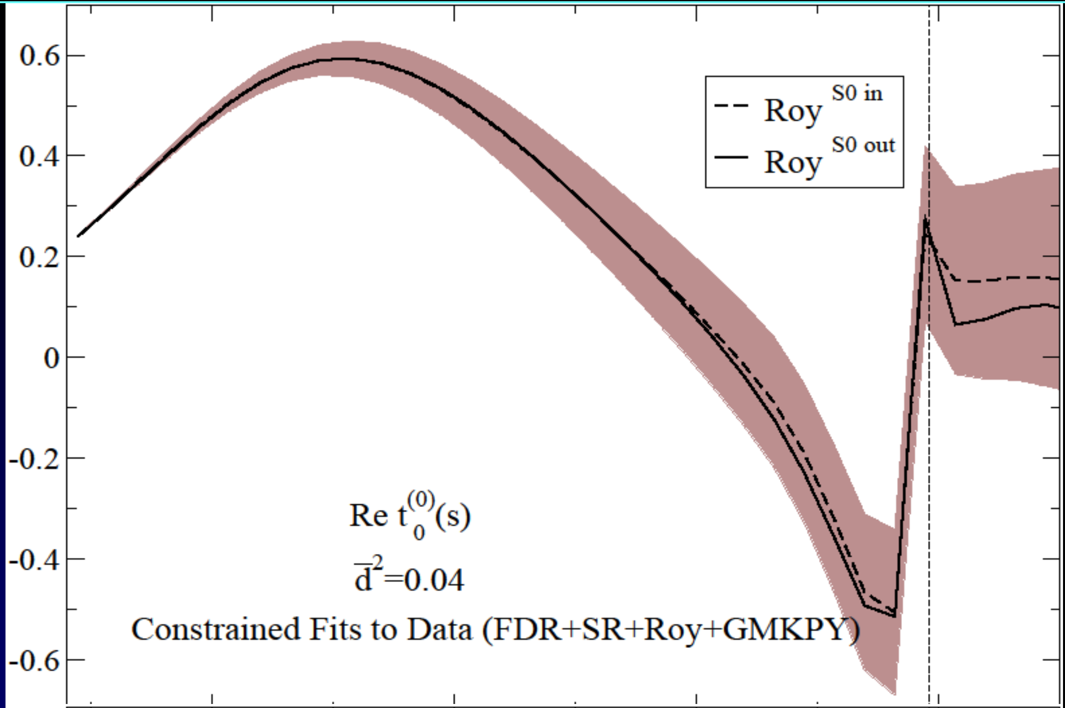


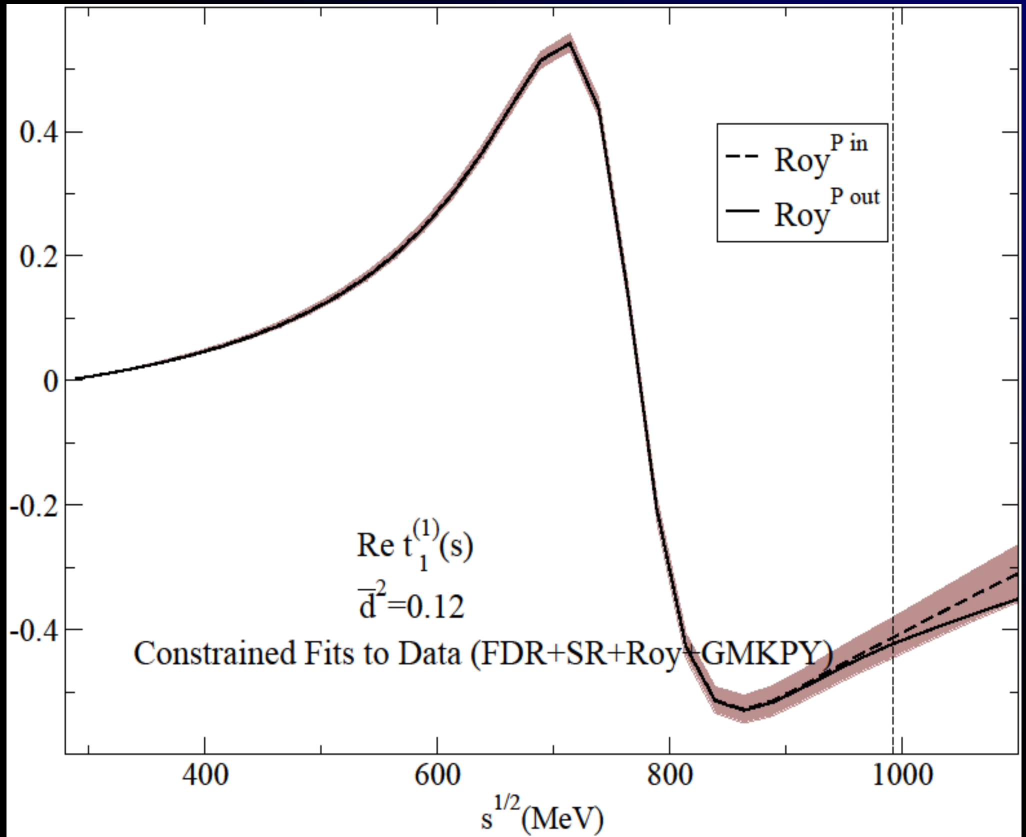
Roy Eqs. for CONSTRAINED fits

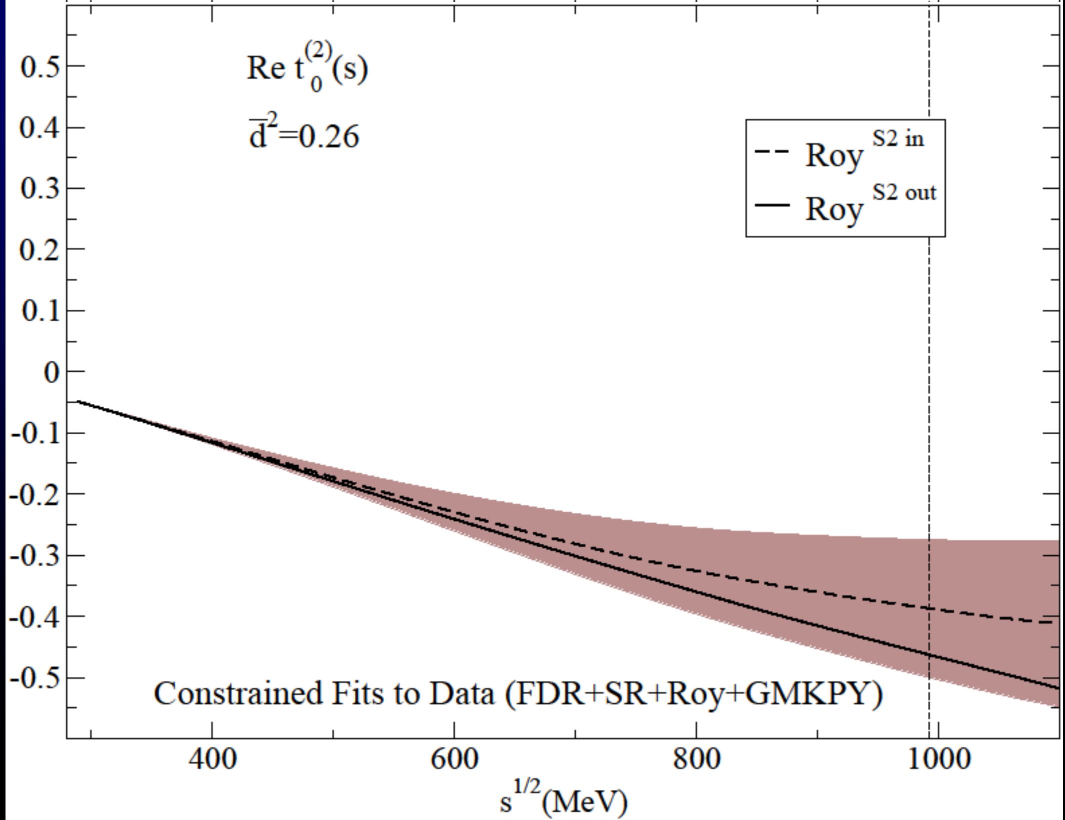
Roy Eqs. averaged d^2

	<932MeV	<1100MeV
S0wave	0.02	0.04
P wave	0.04	0.12
S2 wave	0.21	0.26

VERY GOOD!!!





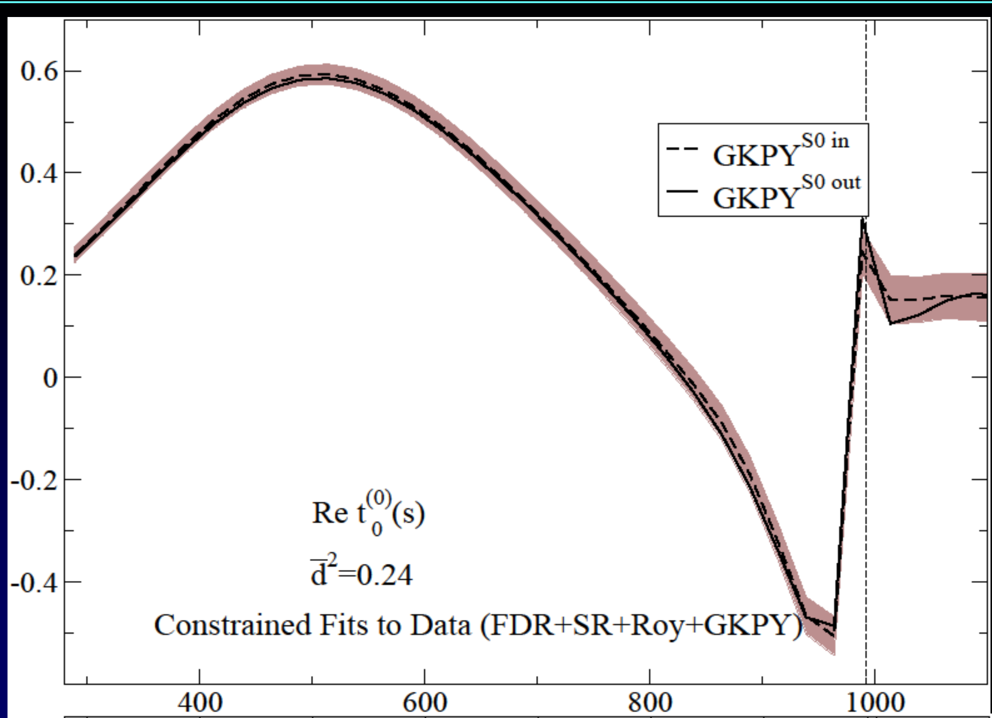


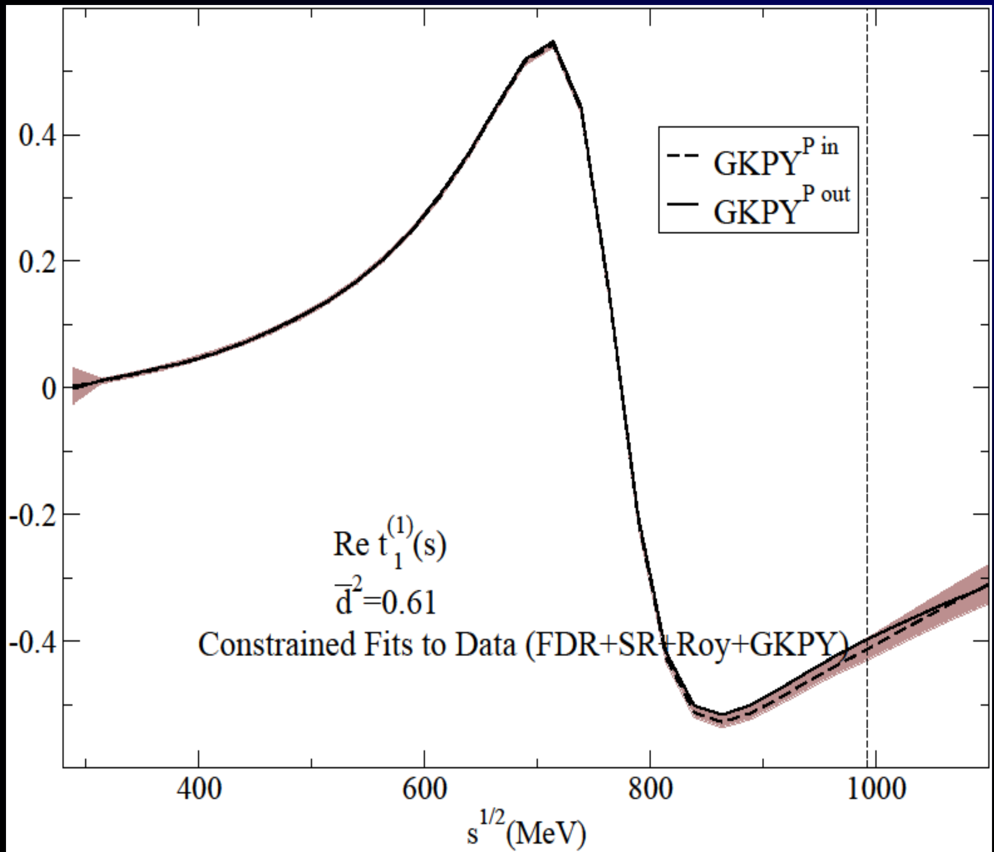
GKPY Eqs. for CONSTRAINED fits

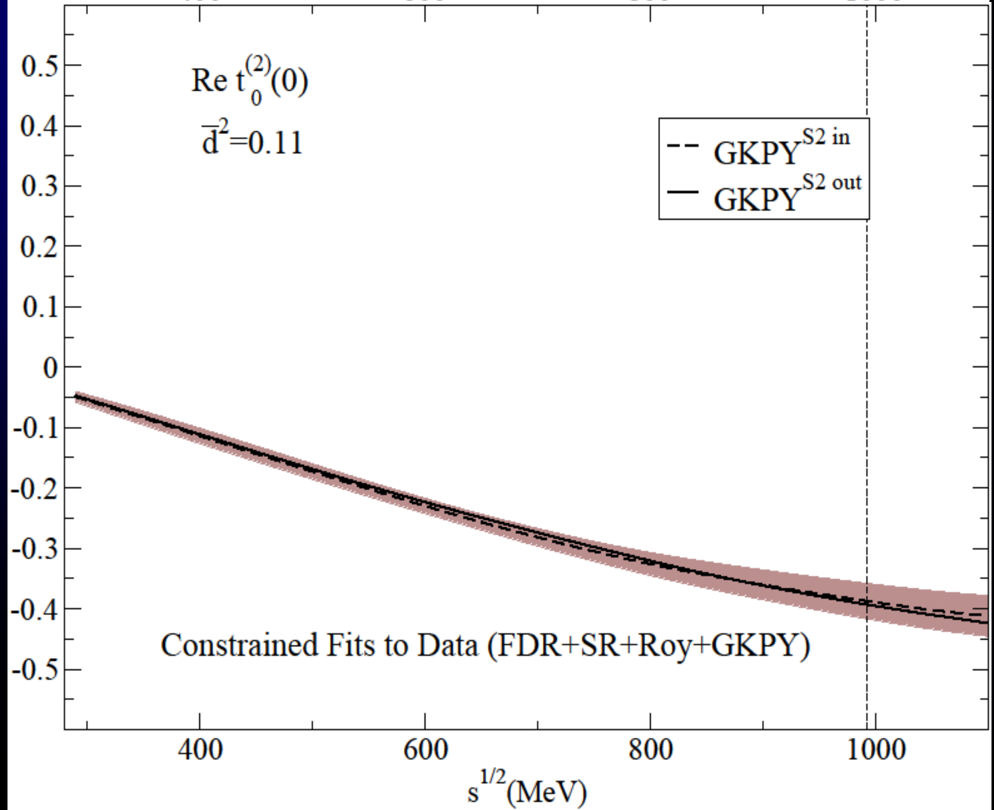
Roy Eqs. averaged d^2

	<932MeV	<1100MeV
S0wave	0.23	0.24
P wave	0.68	0.60
S2 wave	0.12	0.11

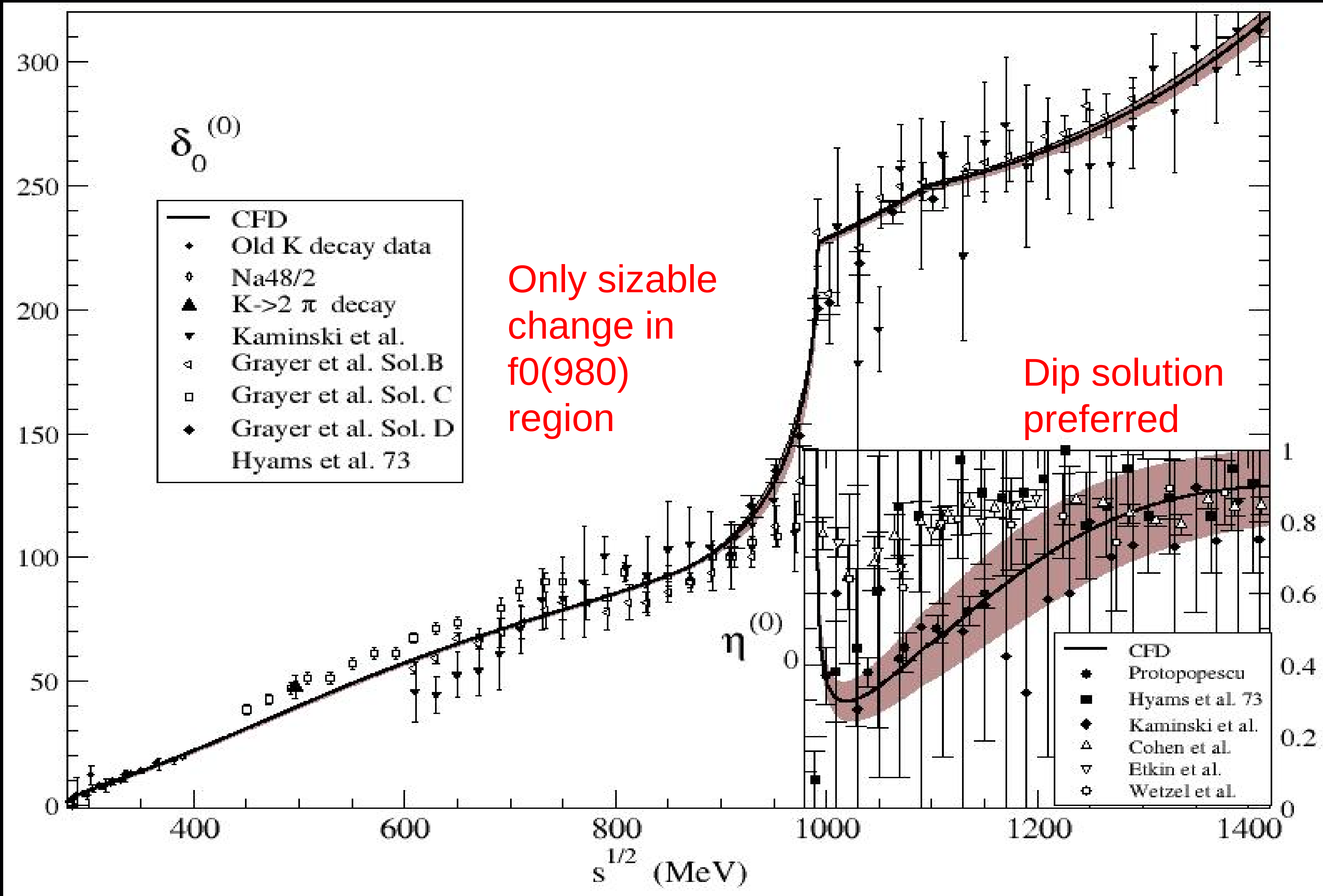
VERY GOOD!!!



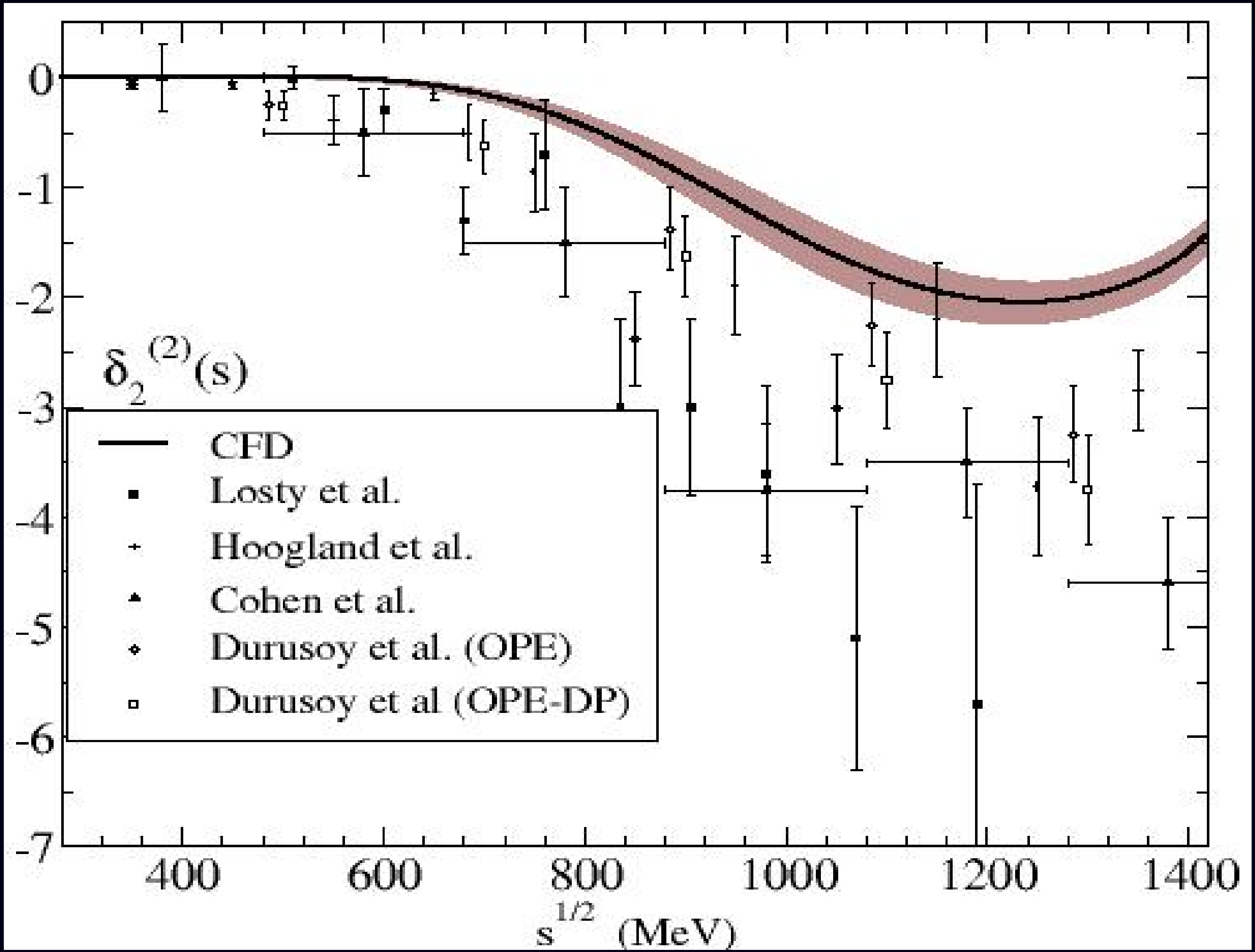




S0 wave: from UFD to CFD



As expected, the wave suffering the largest change is the D2



Apart from S0 and D2, changes in other waves from UFD to CFD is imperceptible

DIP vs NO DIP inelasticity scenarios

Now we find large differences in GKPY S0 wave d^2

UFD

992MeV< e <1100MeV

Dip	6.15
No dip	23.68

CFD

850MeV< e <1050MeV

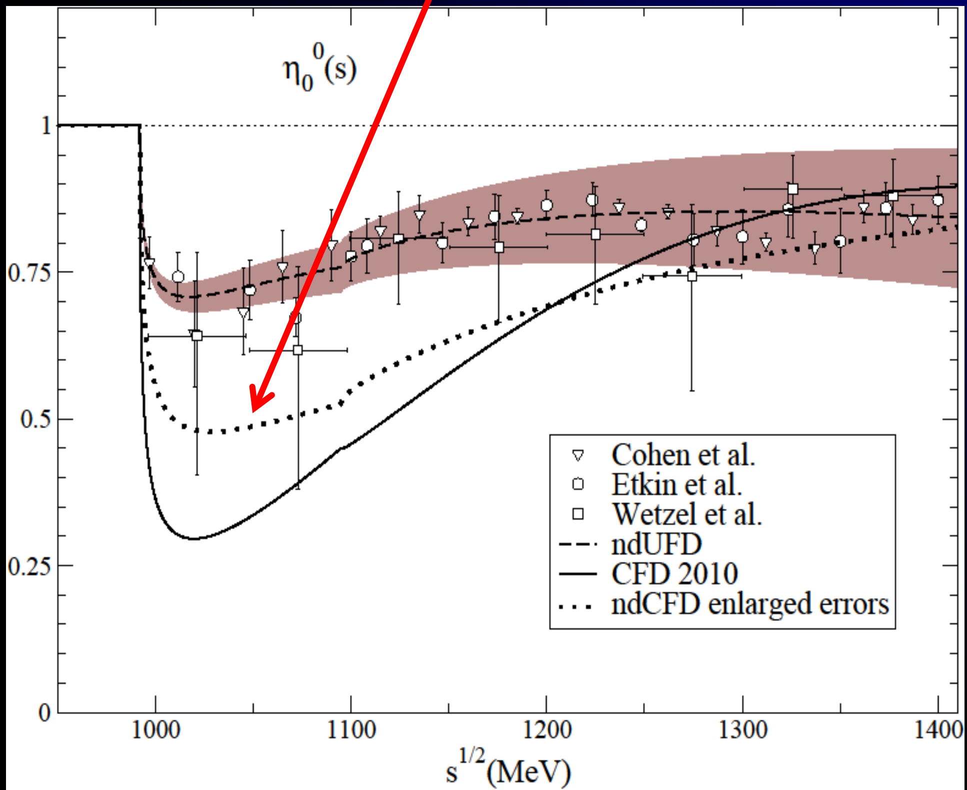
Dip	1.02
No dip	3.49

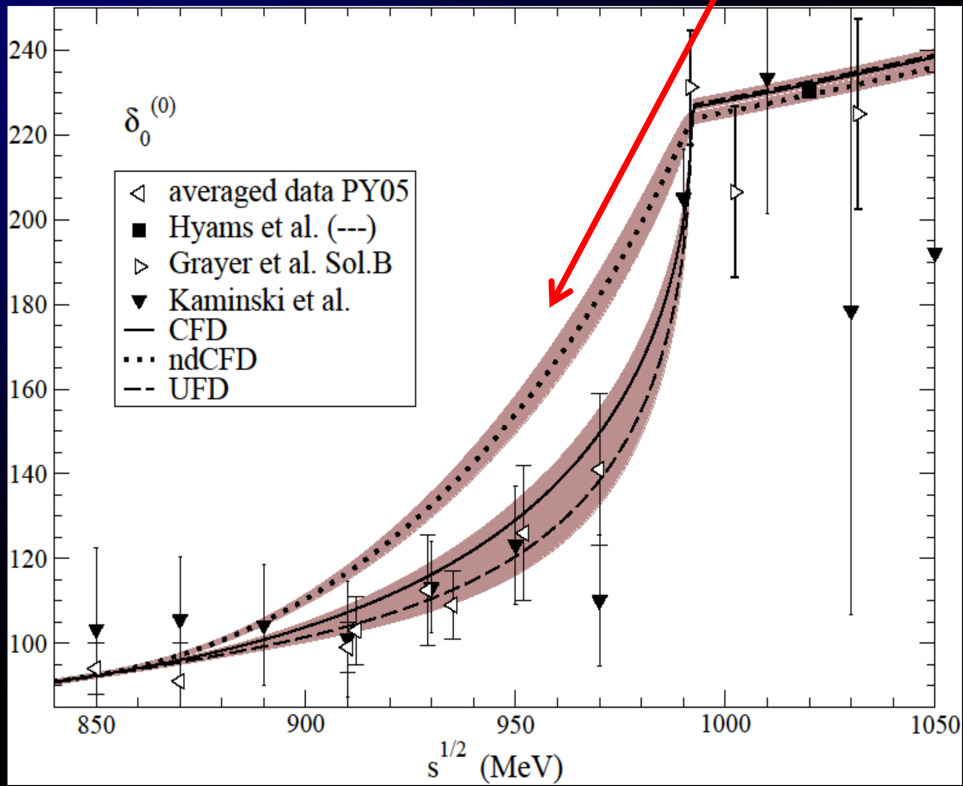
Improvement possible?

No dip (enlarged errors)	1.66
No dip (forced)	2.06

But becomes the “Dip” solution

Other waves worse and data on phase NOT described





Some relevant recent DISPERSIVE POLE Determinations of the f0(980)
(after QCHS-2010, also “according” to PDG)

- GKPY equations = Roy like with one subtraction

García Martín, Kaminski, JRP, Yndurain PRD83,074004 (2011)

Garcia-Martin , Kaminski, JRP, Ruiz de Elvira, PRL107, 072001(2011)

$$(996 \pm 7) - i(25^{+10}_{-6})\text{MeV}$$

- Roy equations

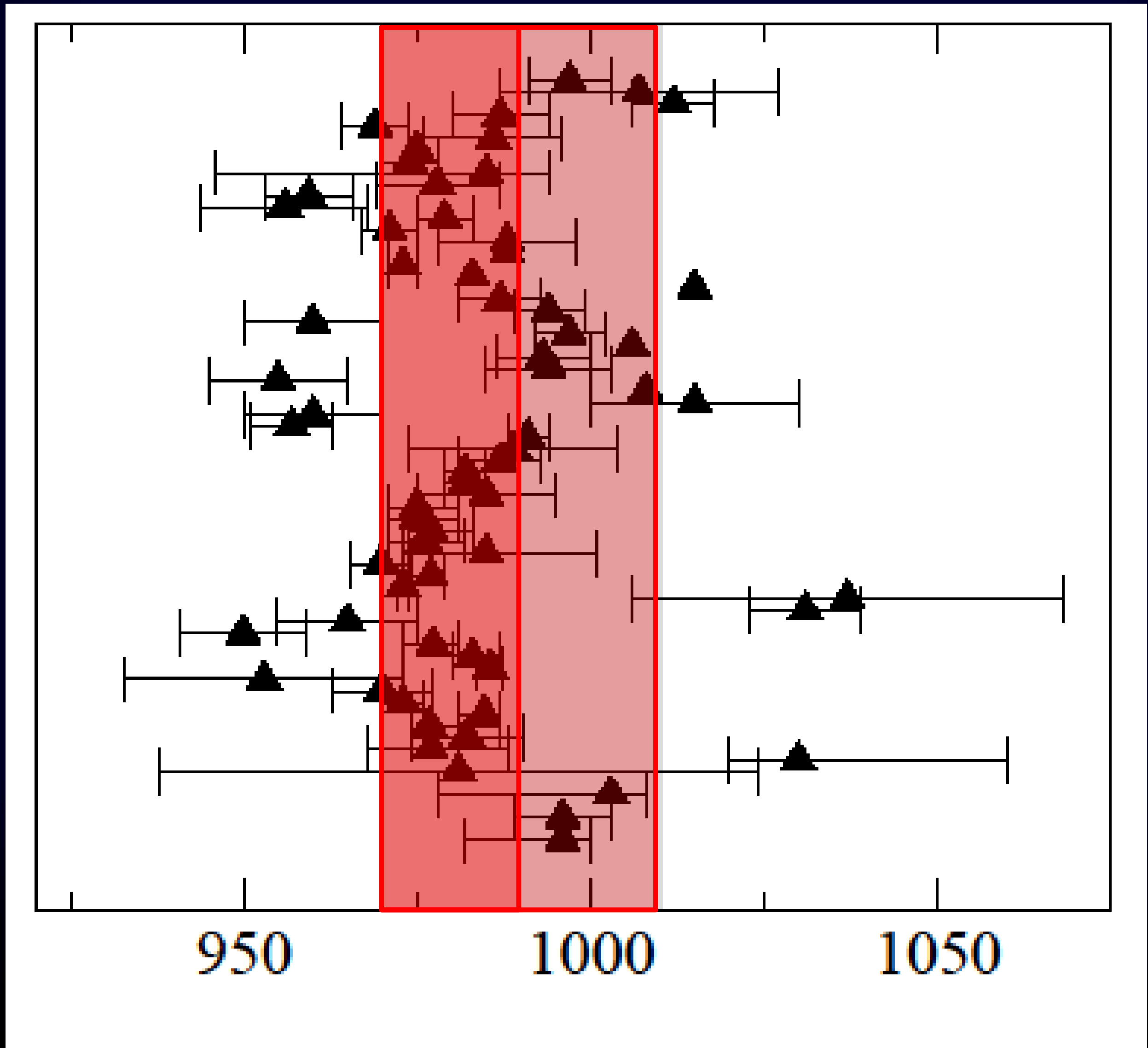
$$(996^{+4}_{-14}) - i(24^{+11}_{-3})\text{MeV}$$

B. Moussallam, Eur. Phys. J. C71, 1814 (2011).

The dip solution favors somewhat higher masses slightly above KK threshold and reconciles widths from production and scattering

Thus, PDG12 made a small correction for the f0(980) mass & more conservative uncertainties

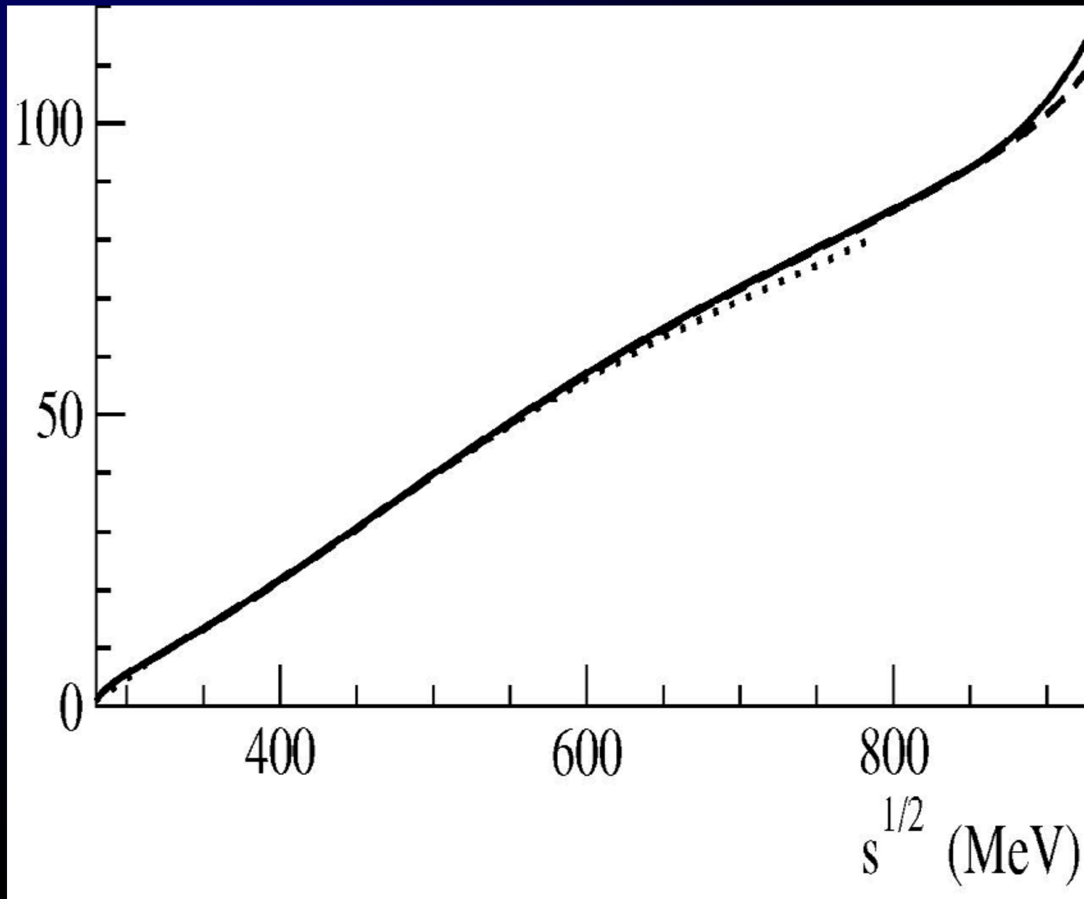
$$M = 980 \pm 10\ \text{MeV} \quad \rightarrow M = 990 \pm 20\ \text{MeV}$$



Other groups (Ananthanarayan, Gasser, Laetwyler, Caprini, Colangelo, Maussallam)
have used Roy Eqs. alone to obtain
SOLUTIONS for the S and P waves below 800 or 1000 MeV,
using the rest as input.

For their most precise results, they use Chiral Perturbation Theory as INPUT
(or universal band)

The results shown so far are
quite consistent with theirs



Our series of works: 2005-2011

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R. García Martín, R. Kaminski, JRP, J. Ruiz de Elvira, F.J. Ynduráin, Phys.Rev. D83 (2011) 074004,
R. García Martín, R. Kaminski, JRP, J. Ruiz de Elvira ,Phys.Rev.Lett. 107 (2011) 072001

Independent and **simple** fits
to data in different channels.
"**Unconstrained Data Fits=UDF**"

↓

Check Dispersion Relations

↓

Impose FDRs, Roy & GKPY Eqs
on data fits
"**Constrained Data Fits CDF**"
Describe data and are consistent with Dispersion relations

For resonance poles: Continuation to complex plane
USING THE DISPERSIVE INTEGRALS

Some relevant Roy-like POLE Determinations

which the PDG took into account in their 2012 σ revision

- **Roy Eqs.** I. Caprini, G. Colangelo, H. Leutwyler PRL97 011601 (2006)

  An S0 Wave solution up to 800 MeV, uses ChPT input

$$(441^{+16}_{-8})-i(272^{+9}_{-12.5})\ \text{MeV}$$

- **GKPY equations = Roy like with one subtraction**

  R. Garcia-Martin , R. Kaminski, JRP, J. Ruiz de Elvira, PRL107, 072001(2011).

  Includes latest NA48/2 **constrained data fit** .One subtraction allows use of data only

  NO ChPT input but good agreement with previous Roy Eqs.+ChPT results.

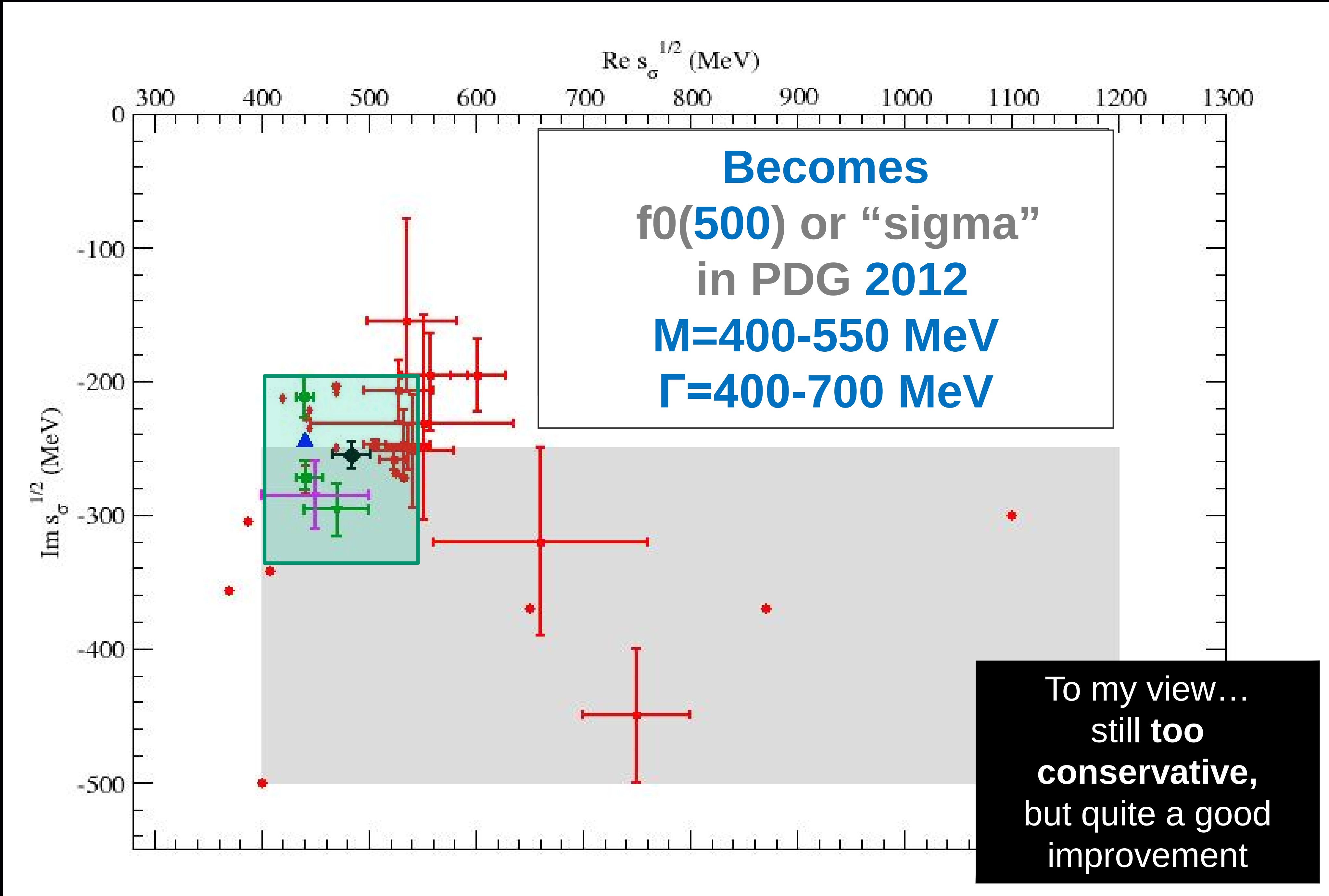
$$(457^{+14}_{-15})-i(279^{+11}_{-7})\text{MeV}$$

- **Roy equations** B. Moussallam, Eur. Phys. J. C71, 1814 (2011).

  An S0 Wave solution up to KK threshold with input from previous Roy Eq. works

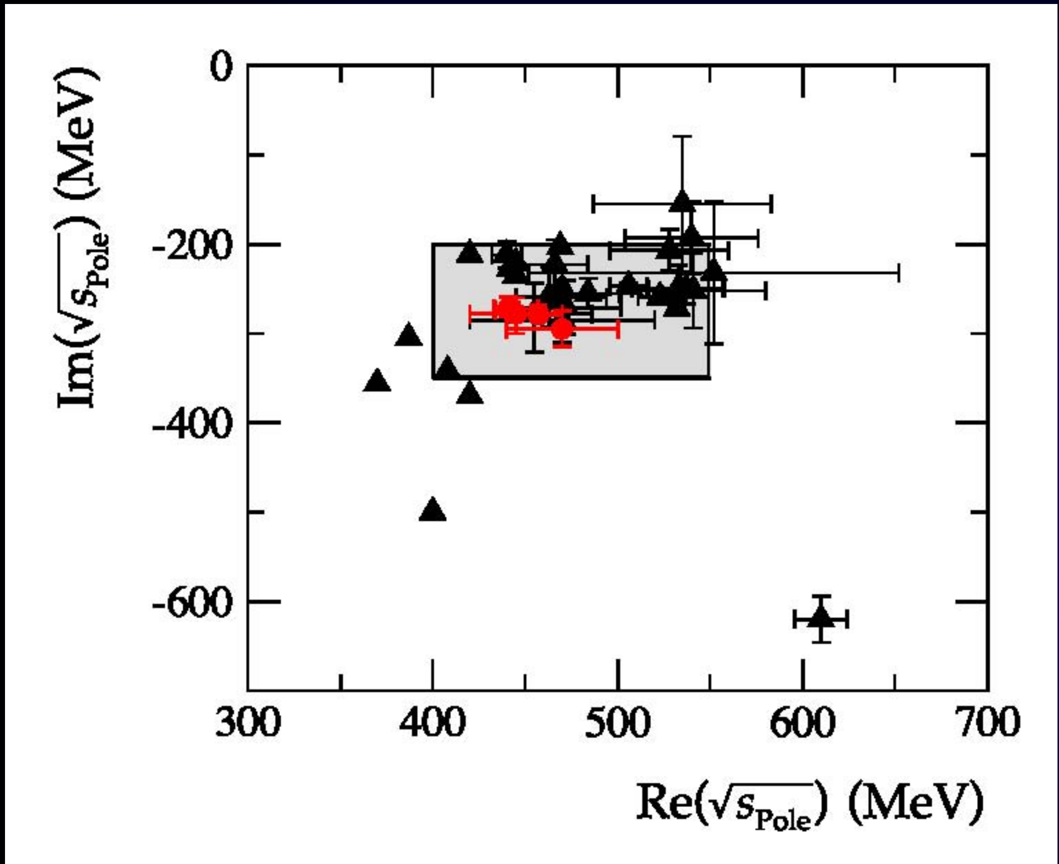
$$(442^{+5}_{-8})-i(274^{+6}_{-5})\text{MeV}$$

DRAMMATIC AND LONG AWAITED CHANGE ON "sigma" RESONANCE @ PDG2012!!



Actually, in the PDG 2017: "Note on scalars"

"One might just consider the most advanced dispersive analyses, Refs. [9–13]. They agree on a pole position close to (450−i 280) MeV."



9. G. Colangelo, J. Gasser, and H. Leutwyler, NPB603, 125 (2001).
10. I. Caprini, G. Colangelo, and H. Leutwyler, PRL 96, 132001 (2006).
11. R. Garcia-Martin , R. Kaminski, JRP, J. Ruiz de Elvira, PRL107, 072001(2011)
12. B. Moussallam, Eur. Phys. J. C71, 1814 (2011)
13. P. Masjuan, J. Ruiz de Elvira, J.J. Sanz-Cillero, PRD90, 097901 (2014).
.

Combining conservatively statistical and systematic uncertainties I estimate:

$$(449^{+22}_{-16})\text{-}i(275\pm 12)\ \text{MeV}$$

JRP, Physics Reports 658-2016-1

This was a long awaited improvement !!!!

Unfortunately, to keep the confusion the PDG still quotes a “Breit-Wigner mass” and width…

I have no words…

But someone else had:

Wovon man night sprechen kann, darüber muß man schweigen

L. Wittgenstein, Tractatus Logico-philosophicus

NEW Global Parameterization

JRP, A.Rodas, J. Ruiz de Elvira. arXiv:1907.13162. To appear in EJPC

- The CFD were very simple, consistent and precise fits. Widely used

- However,
    - they were constructed piece-wise.
    - Only up to 1.4 GeV
    - Only approximation to actual pole values from GKPY

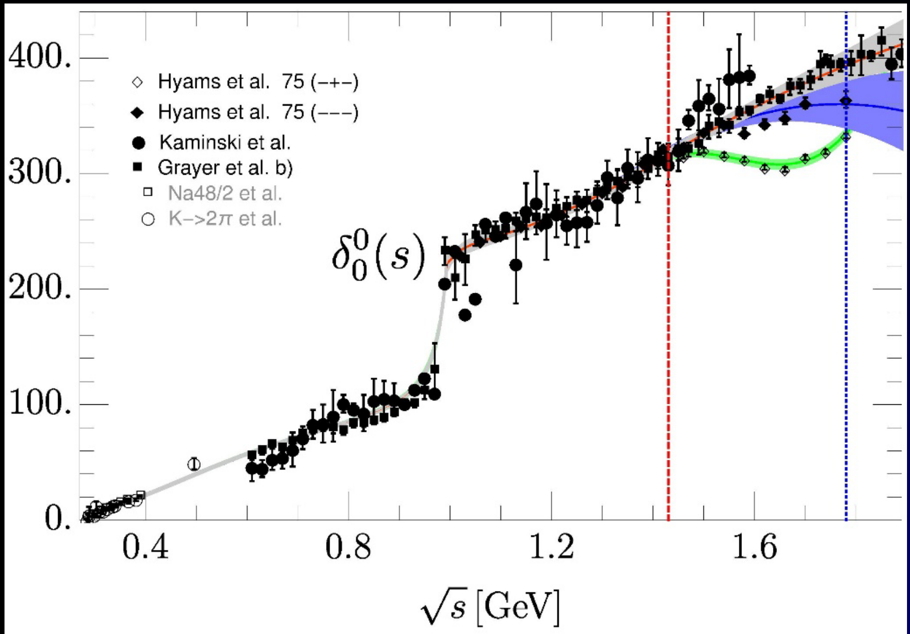
- We have made a **<u>new global parameterization</u>** os S0 and P waves.
    - Not piece-wise
    - Consistent with CFD on the real axis
    - Consistent with GKPY in the complex plane (Lehmann Ellipse)
    - Consistent with GKPY up to 1.1 GeV and FDRs up to 1.4 GeV
    - Consistent pole positions for $f_0(500)$, $f_0(980)$ and $\rho(770)$
    - Fits data up to 2 GeV but 3 different solutions.
    - Simple expressions easy to implement

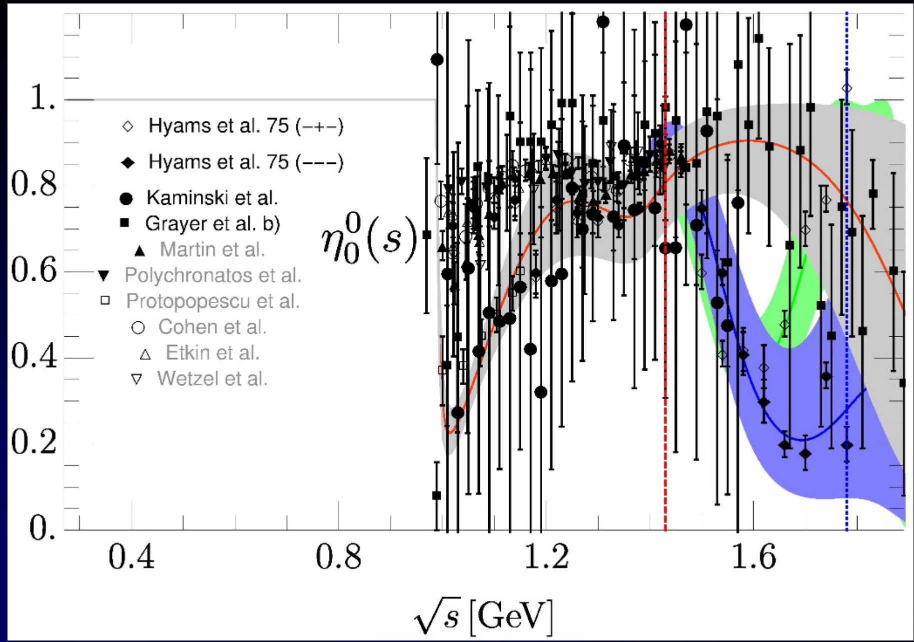
NEW

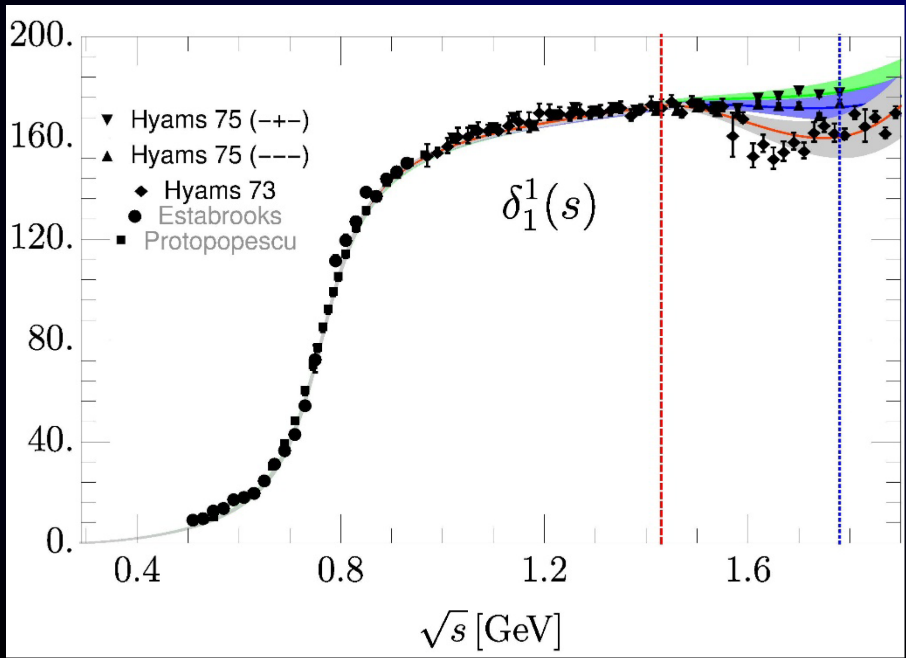
NEW Global Parameterization

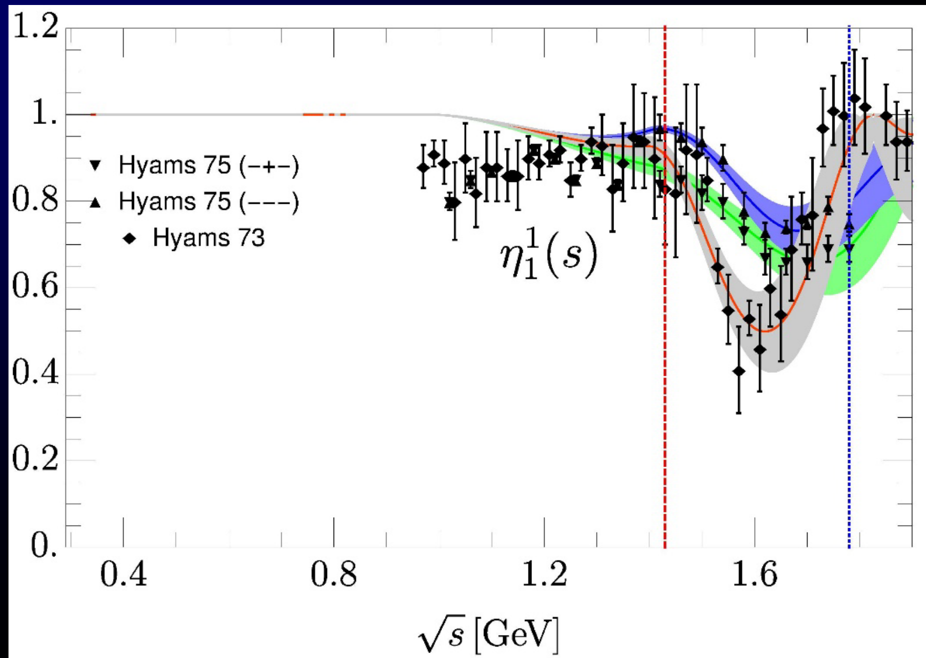
JRP, A.Rodas, J. Ruiz de Elvira. arXiv:1907.13162. To appear in EJPC

NEW









Simple Unconstrained Fits to πK partial-wave Data (UFD).
Estimation of statistical and SYSTEMATIC errors

Forward Dispersion Relations:
Left cut easy to rewrite
Relate amplitudes, not partial waves
Not direct access to pole

- As πK checks: Small inconsistencies.

Forward dispersion relations for K π scattering.

Since interested in the resonance region, we use minimal number of subtractions

Defining the s↔u symmetric and anti-symmetric amplitudes at t=0

$$T^{+}(s) = \frac{T^{1/2}(s) + 2T^{3/2}(s)}{3} = \frac{T^{I_t=0}(s)}{\sqrt{6}},$$
$$T^{-}(s) = \frac{T^{1/2}(s) - T^{3/2}(s)}{3} = \frac{T^{I_t=1}(s)}{2}.$$

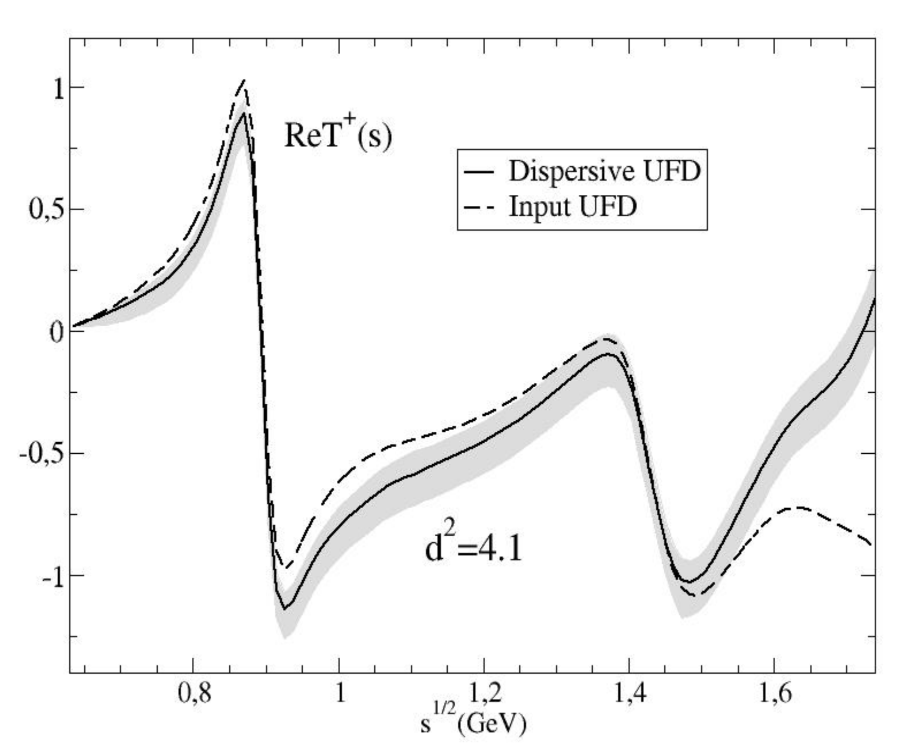
We need one subtraction for the symmetric amplitude

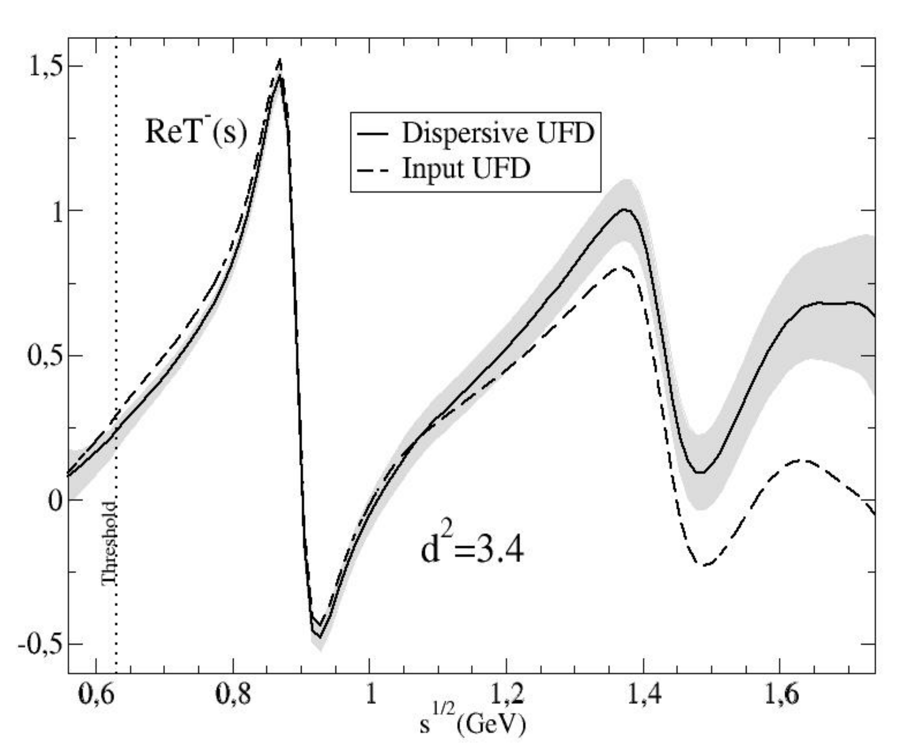
$$\mathrm{Re}T^{+}(s) = T^{+}(s_{\mathrm{th}}) + \frac{(s - s_{\mathrm{th}})}{\pi} P \int_{s_{\mathrm{th}}}^{\infty} ds' \left[\frac{\mathrm{Im}T^{+}(s')}{(s' - s)(s' - s_{\mathrm{th}})} - \frac{\mathrm{Im}T^{+}(s')}{(s' + s - 2\Sigma_{\pi K})(s' + s_{\mathrm{th}} - 2\Sigma_{\pi K})} \right],$$

And none for the antisymmetric

$$\mathrm{Re}T^{-}(s) = \frac{(2s - 2\Sigma_{\pi K})}{\pi} P \int_{s_{\mathrm{th}}}^{\infty} ds' \frac{\mathrm{Im}T^{-}(s')}{(s' - s)(s' + s - 2\Sigma_{\pi K})}.$$

where $\Sigma_{\pi K} = m_\pi^2 + m_K^2$





Forward Dispersion Relation analysis of πK scattering DATA up to 1.6 GeV

(**not a solution** of dispersión relations, but a constrained fit)

A.Rodas & JRP, PRD93,074025 (2016)

First observation:
Forward Dispersion relations
Not well satisfied by data
Particularly at high energies

So we use
Forward Dispersion Relations
as CONSTRAINTS on fits

Our Dispersive/Analytic Approach for πK and strange resonances

Simple Unconstrained Fits to πK partial-wave Data (UFD).
Estimation of statistical and SYSTEMATIC errors

Forward Dispersion Relations:
Left cut easy to rewrite
Relate amplitudes, not partial waves
Not direct access to pole

- As πK checks: Small inconsistencies.
- As constraints:
  πK consistent fits up to 1.6 GeV JRP, A.Rodas,Phys.Rev. D93 (2016)

How well Forward Dispersion Relations are satisfied by unconstrained fits

Every 22 MeV calculate the difference between both sides of the DR /uncertainty

Define an averaged χ^2 over these points, that we call d^2

> d^2 close to 1 means that the relation is well satisfied
>
> d^2>> 1 means the data set is inconsistent with the relation.

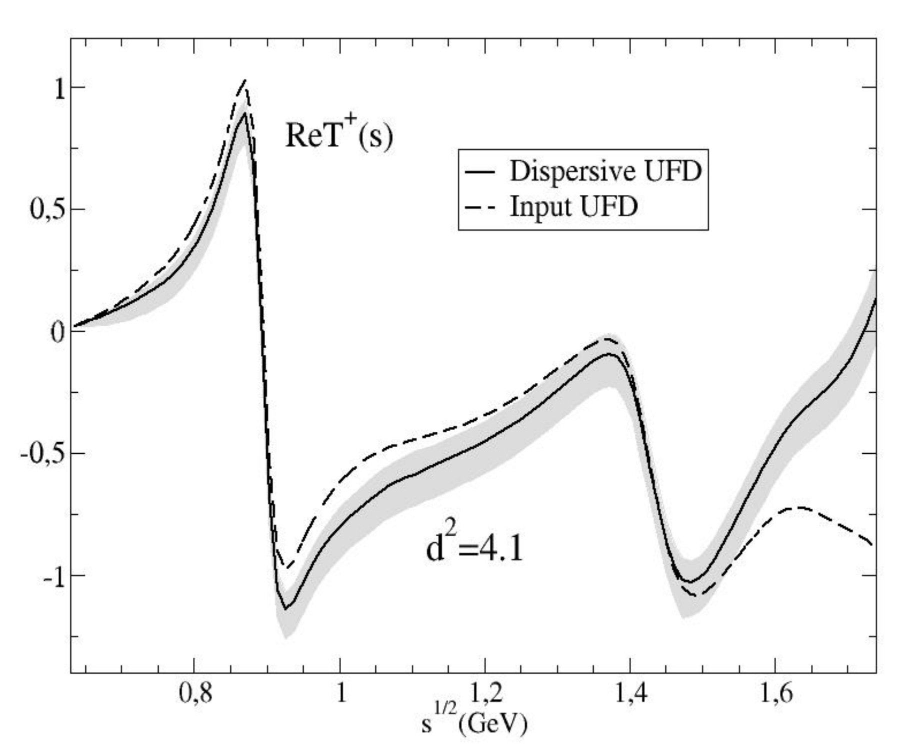
This can be used to check DR

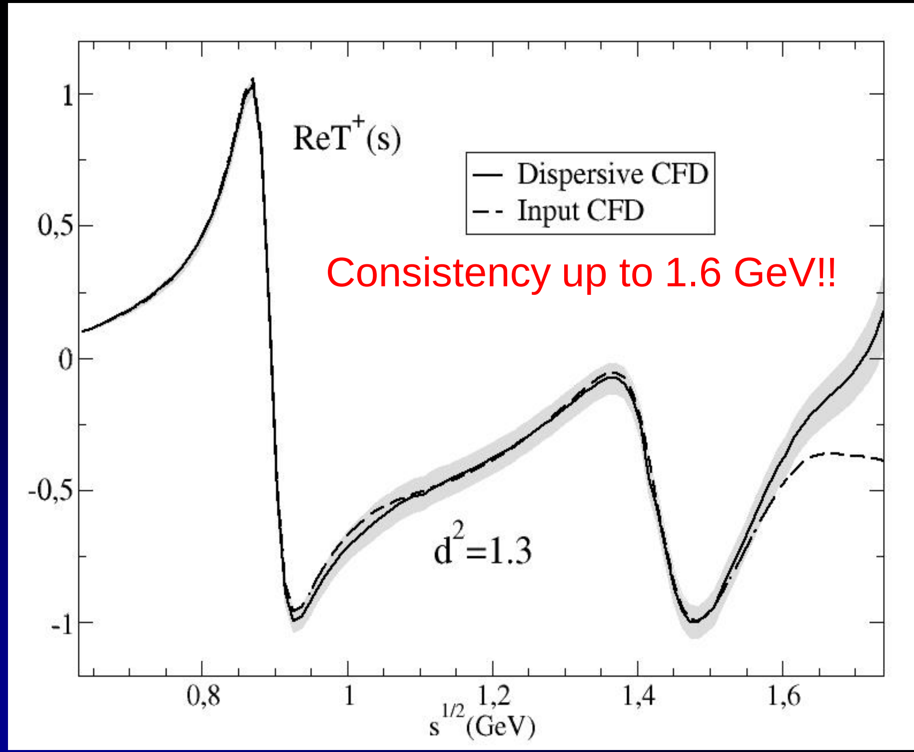
To obtain CONSTRAINED FITS TO DATA (CFD) we minimize:

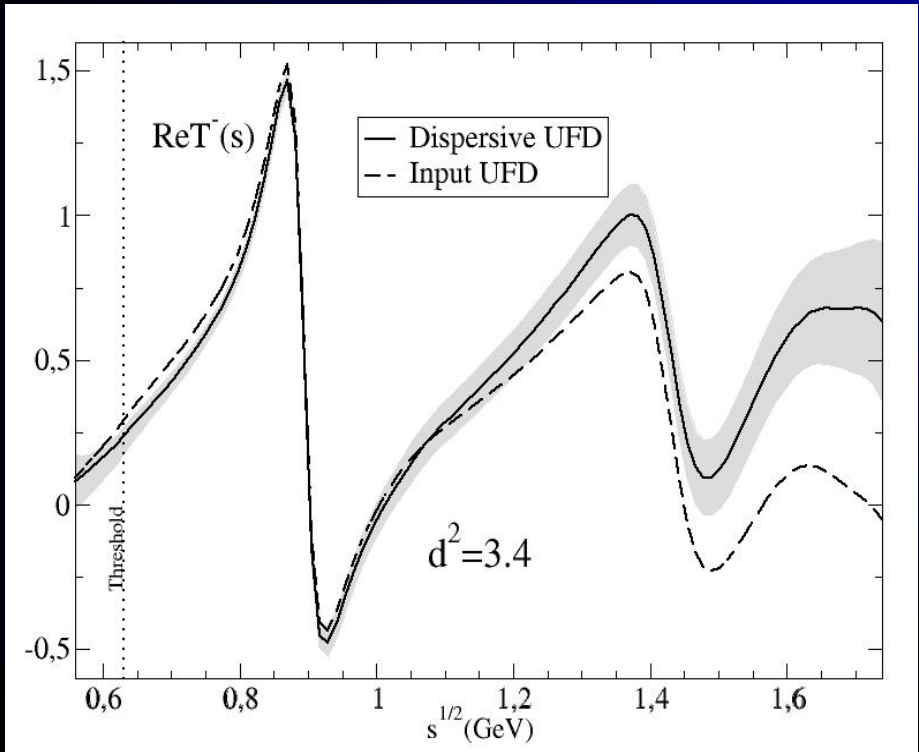
$$W^2\left(d^2_{T^+}+d^2_{T^-}\right)+\sum_{I=\frac{1}{2},\frac{3}{2}}\left(\frac{\Delta_I}{\delta\Delta_I}\right)^2+\sum_k\left(\frac{p_k^{UFD}-p_k}{\delta p_k^{UFD}}\right)^2,$$

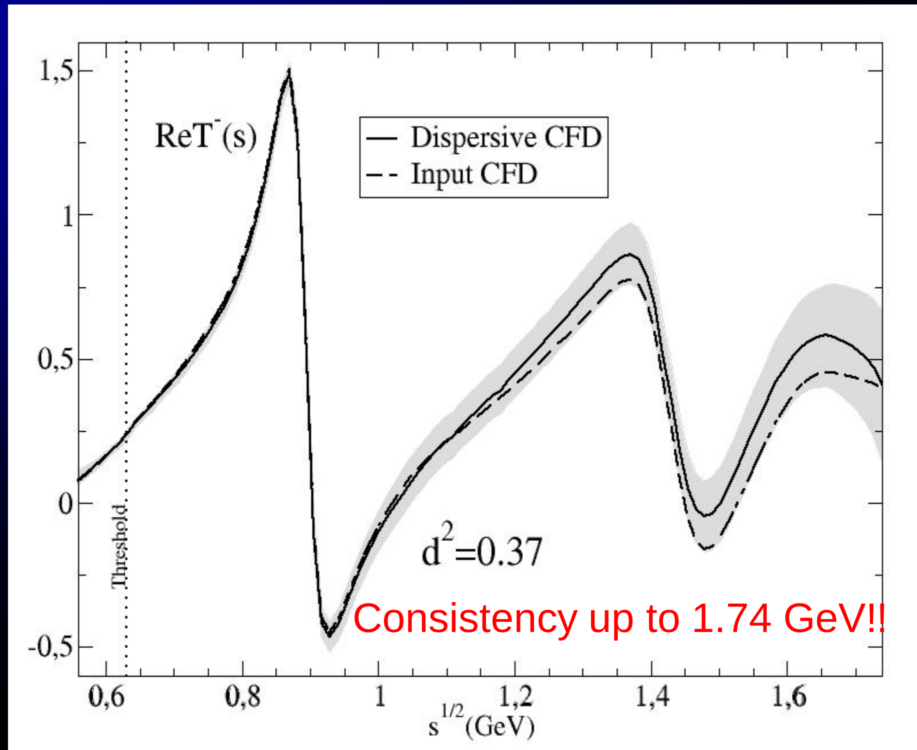
2 FDR's — Sum Rules threshold — Parameters of the unconstrained data fits

W roughly counts the number
of effective degrees of freedom
(sometimes we add weight on certain energy regions)


ReT⁺(s)
Dispersive UFD
Input UFD
d²=4.1
s^1/2 (GeV)


ReT⁺(s)
Dispersive CFD
Input CFD
Consistency up to 1.6 GeV!!
d²=1.3
s^1/2 (GeV)


ReT⁻(s)
Dispersive UFD
Input UFD
d²=3.4
Threshold
s^1/2 (GeV)

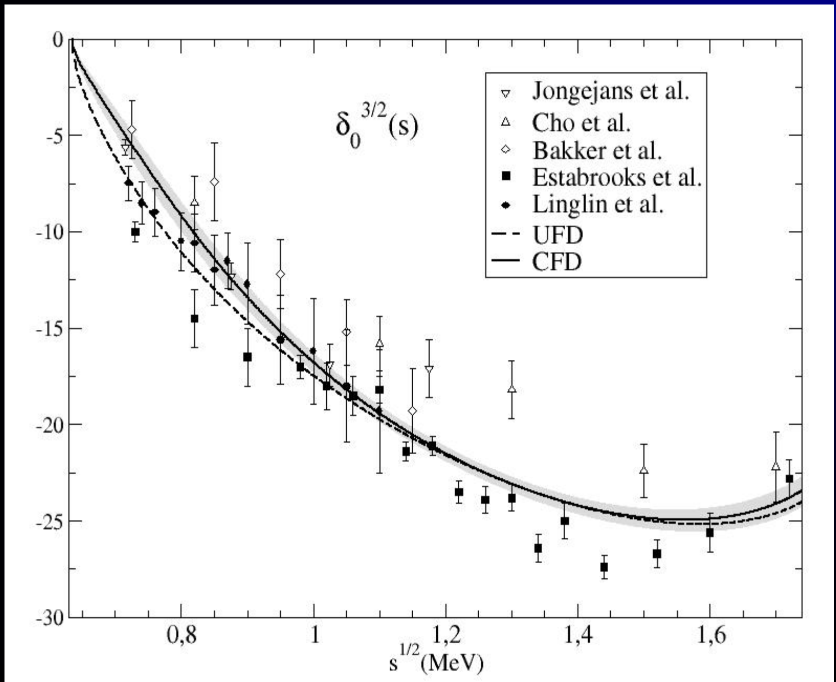

ReT⁻(s)
Dispersive CFD
Input CFD
d²=0.37
Consistency up to 1.74 GeV!!
Threshold
s^1/2 (GeV)

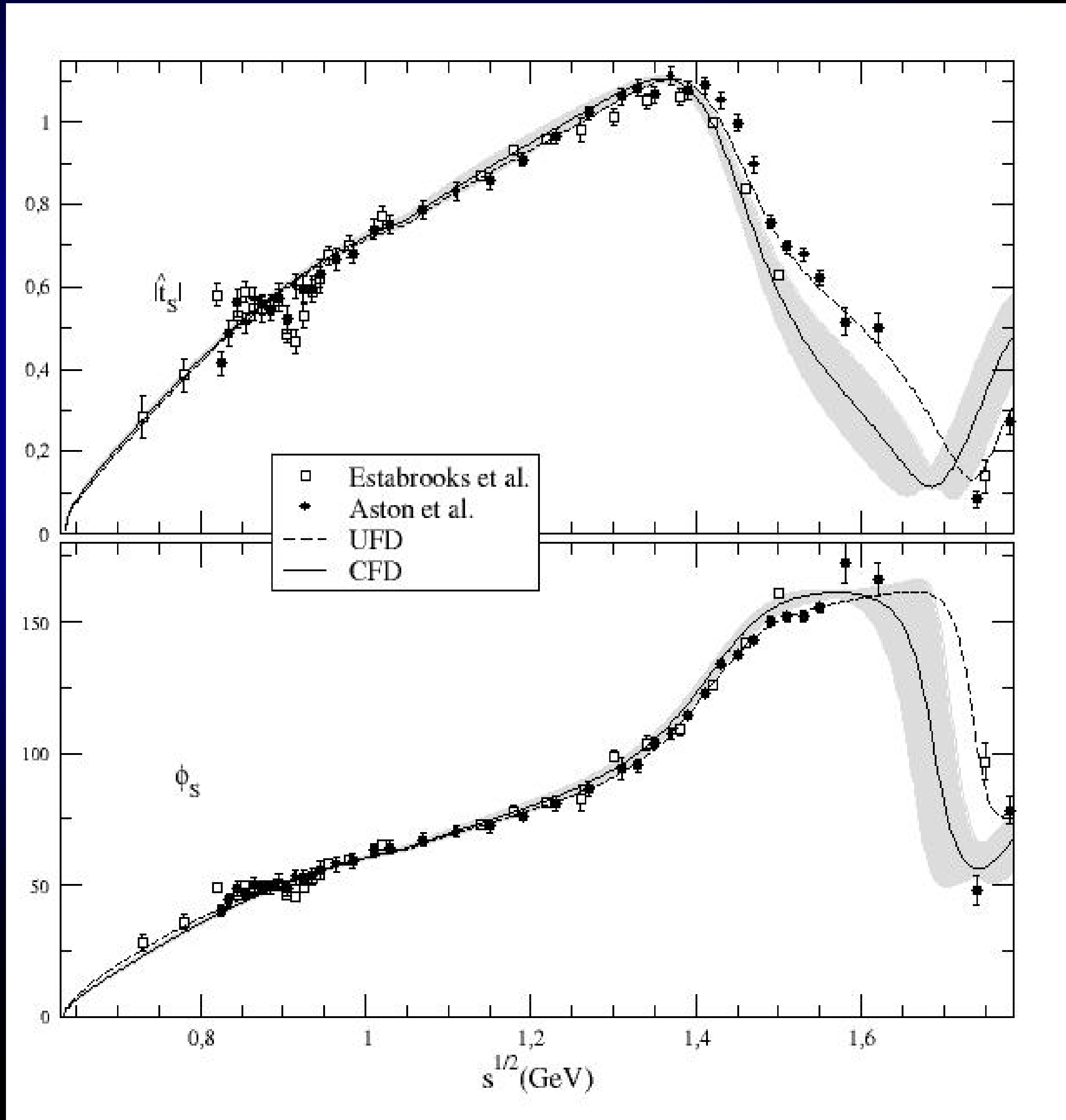
From Unconstrained (UFD) to Constrained Fits to data (CFD)

S-waves. The most interesting for the K_0^* resonances

Largest changes from UFD to CFD

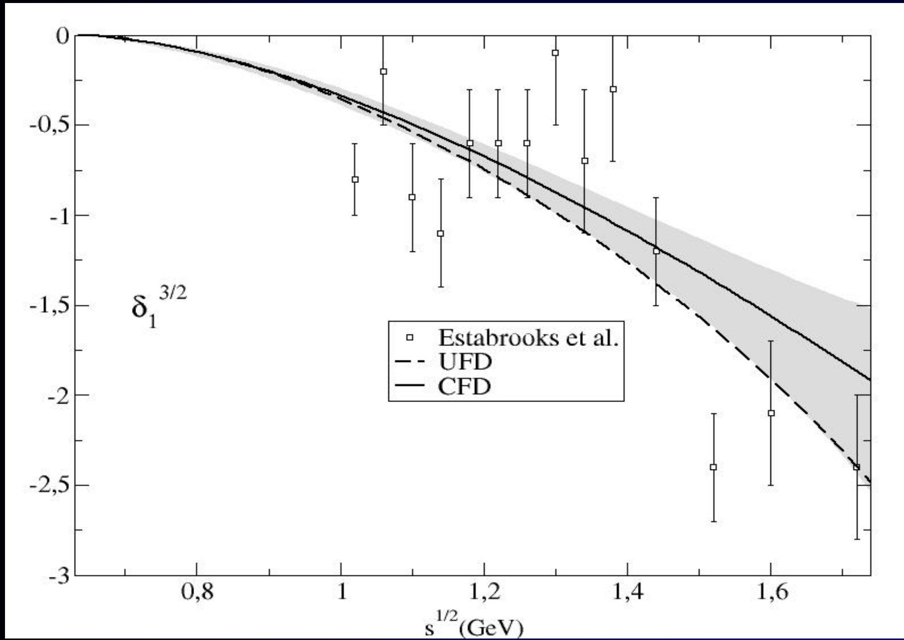
at higher energies

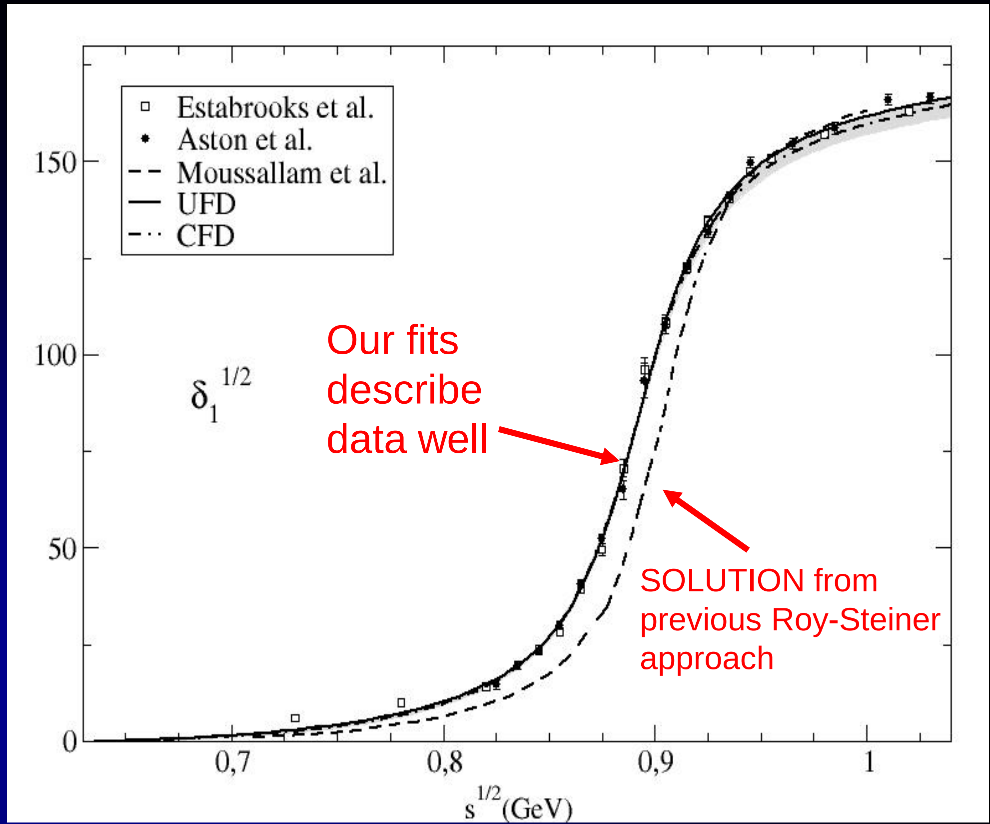


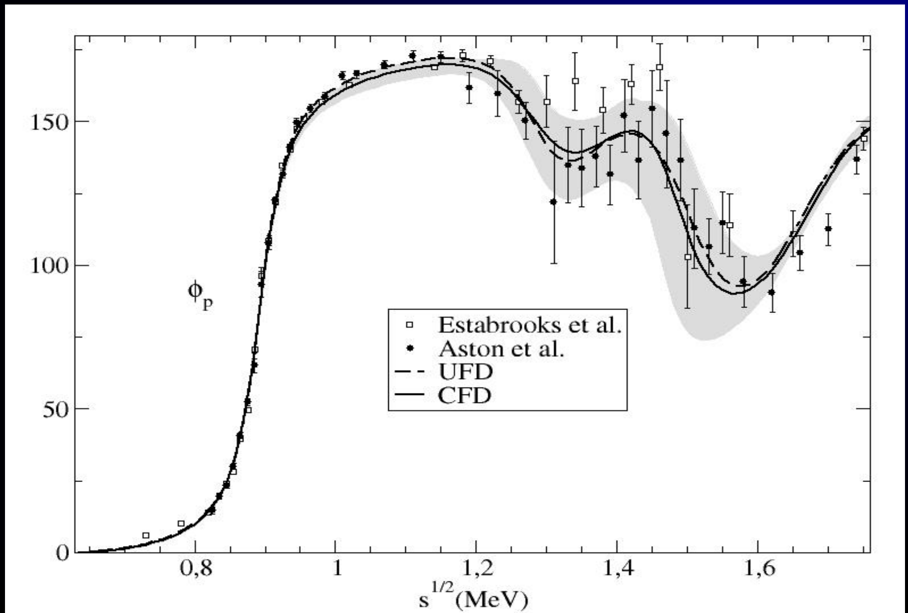


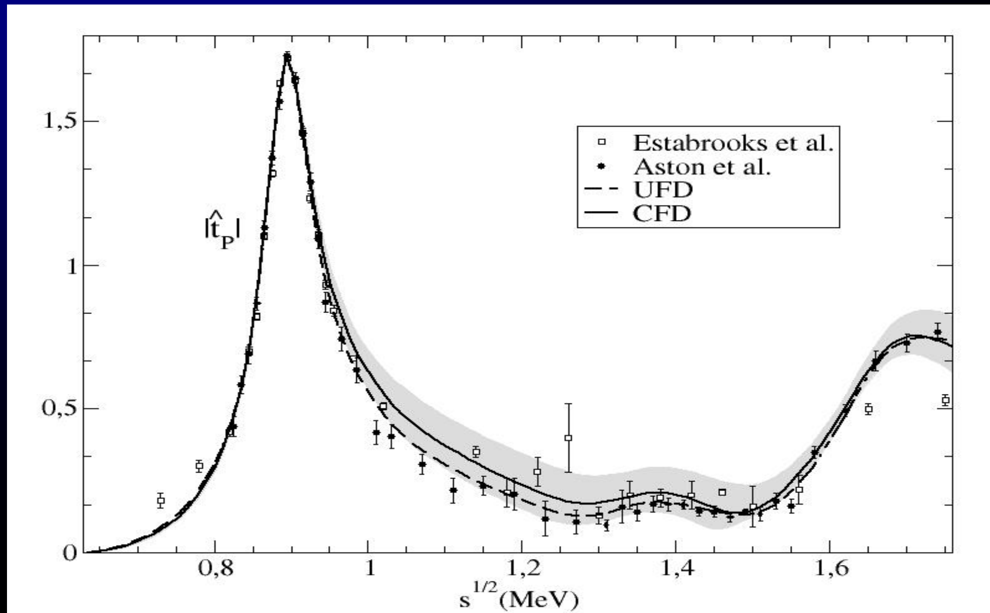
From Unconstrained (UFD) to Constrained Fits to data (CFD)

P-waves: Small changes

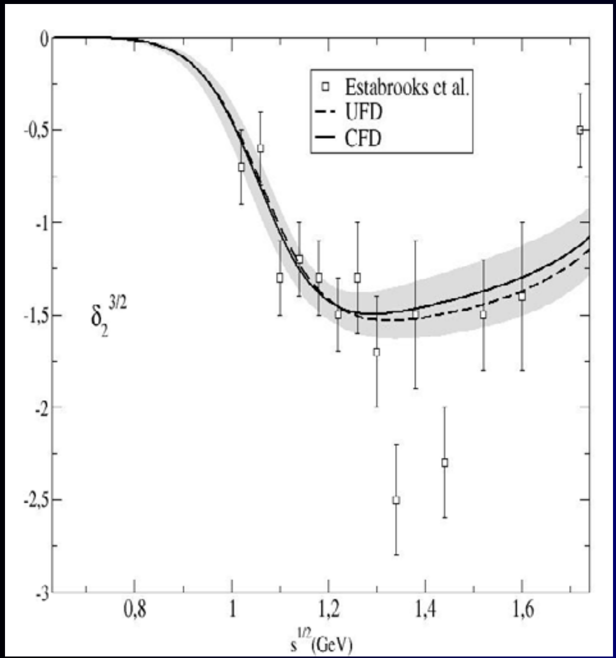


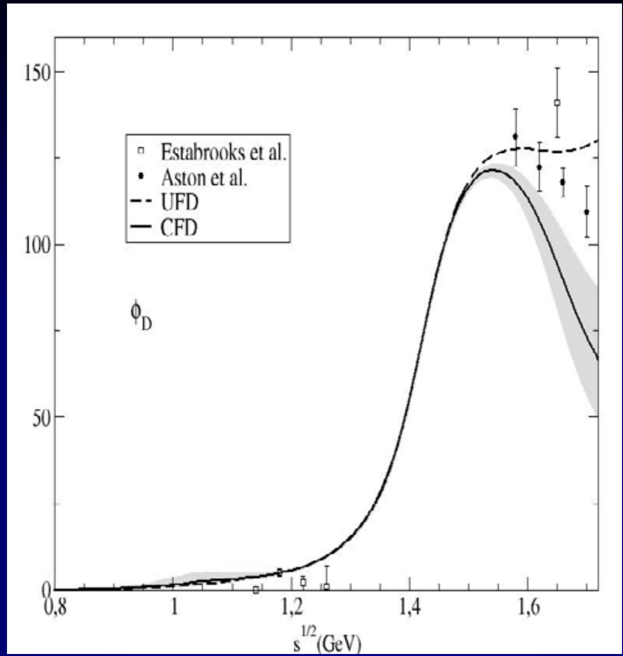


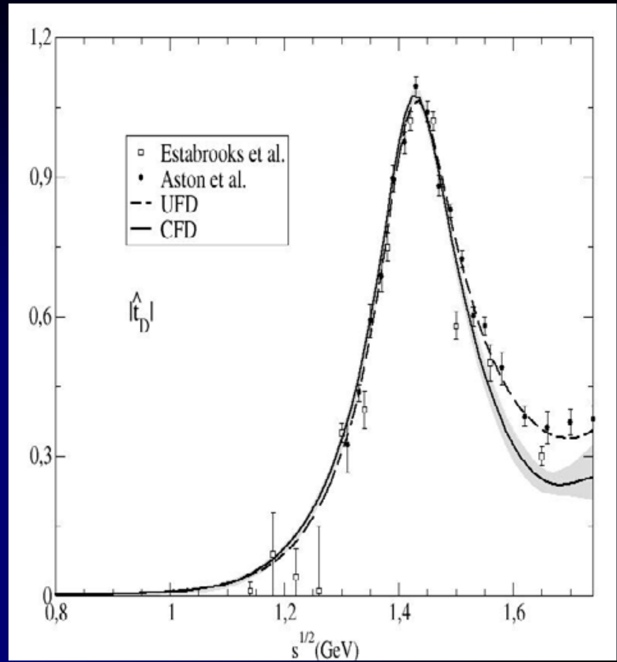




D-waves: Largest changes of all, but at very high energies

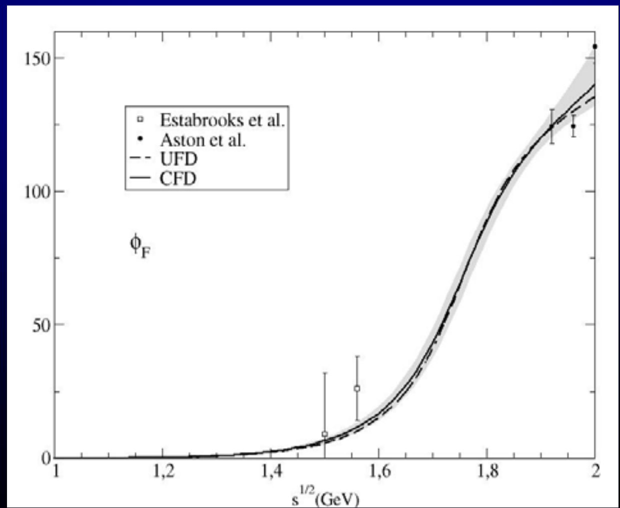


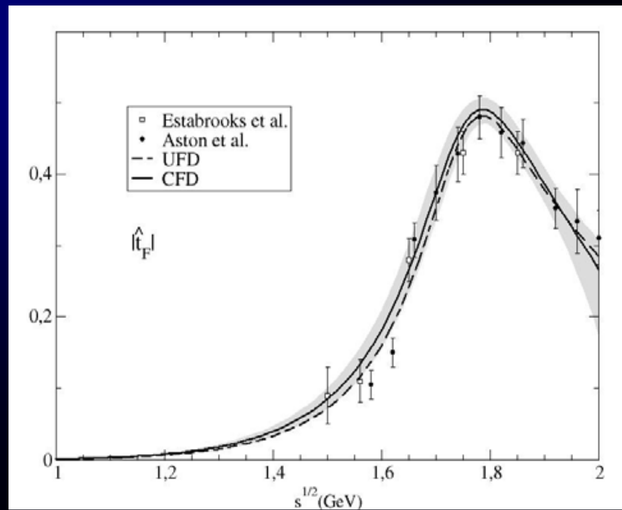




F-waves:

Imperceptible changes





Regge parameterizations allowed to vary: Only πK-ρ residue changes by 1.4 deviations

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Simple Unconstrained Fits to πK partial-wave Data (UFD).
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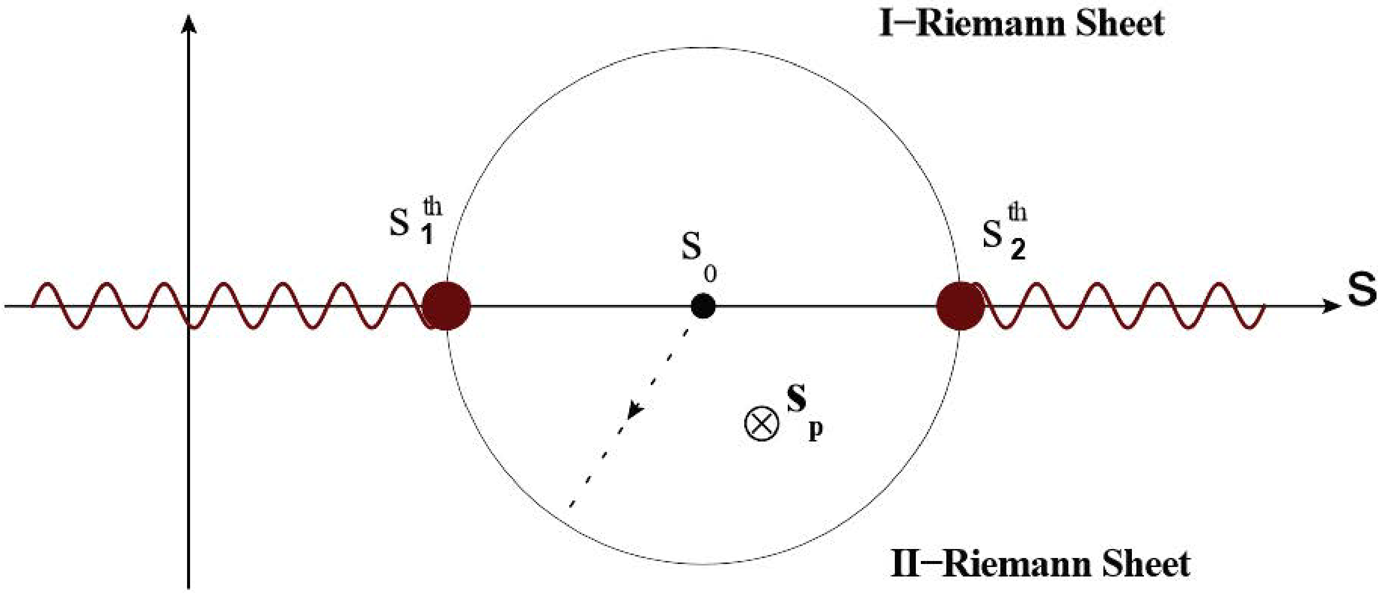
Strange resonance poles from CFD: Using Padé sequences

JRP, A.Rodas & J. Ruiz de Elvira. Eur. Phys. J. C (2017)

Almost model independent: Does not assume any particular functional form (but local determination)

Based on previous works by P.Masjuan, J.J. Sanz Cillero, I. Caprini, J.Ruiz de ELvira

- The method is suitable for the calculation of both elastic and inelastic resonances.
- The Padé sequence gives us the continuation to the continuous Riemann Sheet.
- We take care of the calculation of the errors. Apart from the experimental and systematic errors of each parameterization we also include different fits.



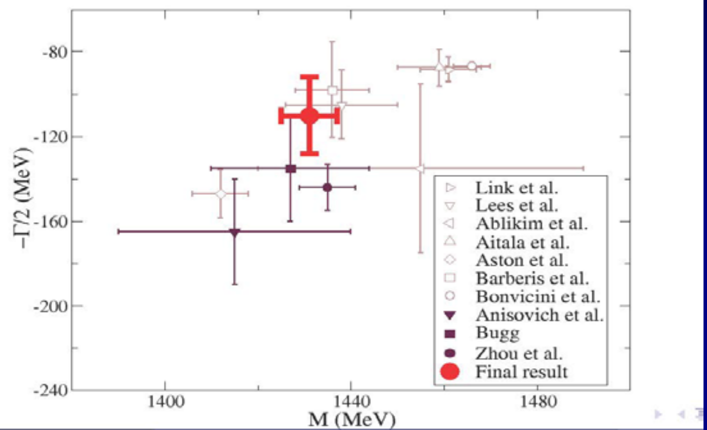
Strange resonance poles from CFD: Using Padé sequences

JRP, A.Rodas & J. Ruiz de Elvira. Eur. Phys. J. C (2017)

The method can be used for inelastic resonances too. Provides resonance parameters WITHOUT ASSUMING SPECIFIC FUNCTIONAL FORM

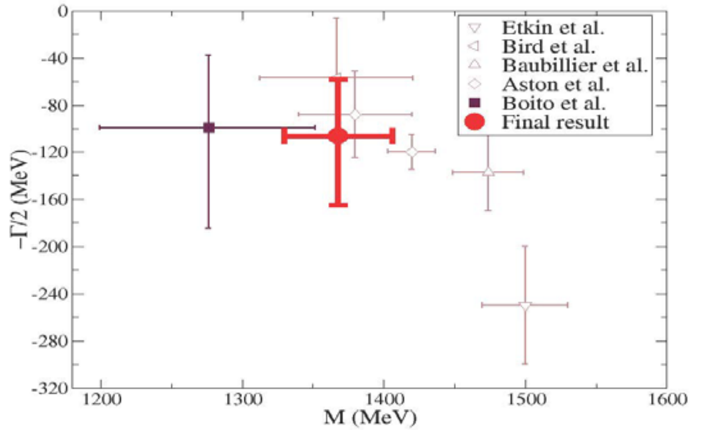
- For the $K_0^*(1430)$ we find

$$\sqrt{s_p} = (1431 \pm 6) - i(110 \pm 19)MeV$$
$$\sqrt{s_p} = (1425 \pm 50) - i(135 \pm 40)MeV(PDG)$$



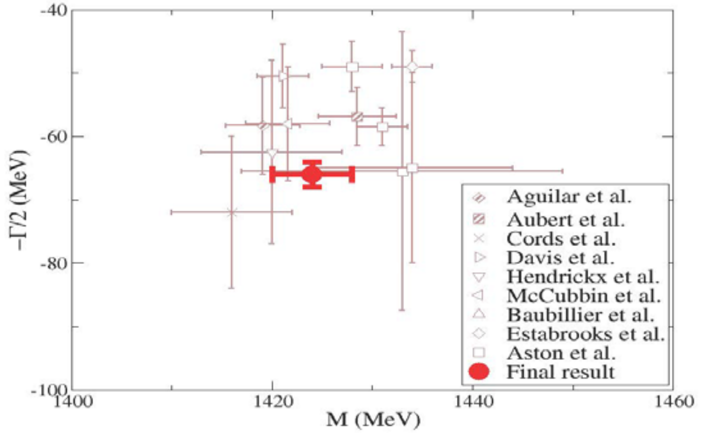
- For the $K_1^*(1410)$ we find

$$\sqrt{s_p} = (1368 \pm 38) - i(106^{+48}_{-59})MeV$$
$$\sqrt{s_p} = (1414 \pm 15) - i(116 \pm 10)MeV(PDG)$$



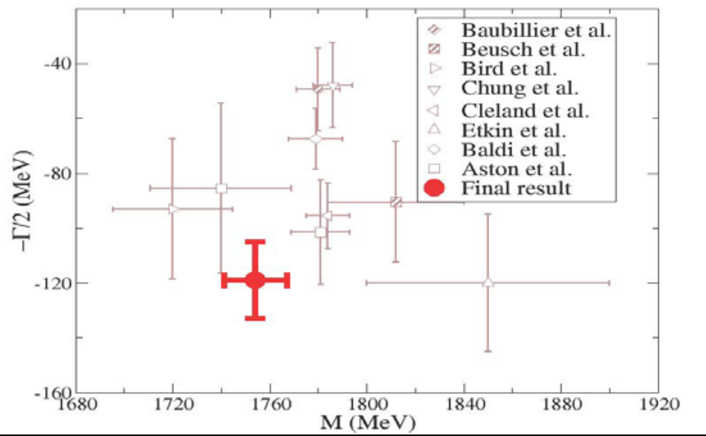
- For the $K_2^*(1430)$ we find

$$\sqrt{s_p} = (1424 \pm 4) - i(66 \pm 2)MeV$$
$$\sqrt{s_p} = (1432.4 \pm 1.3) - i(55 \pm 3)MeV(PDG)$$



- For the $K_3^*(1780)$ we find

$$\sqrt{s_p} = (1754 \pm 13) - i(119 \pm 14)MeV$$
$$\sqrt{s_p} = (1776 \pm 7) - i(80 \pm 11)MeV(PDG)$$



Kappa pole analytic determinations from constrained fits

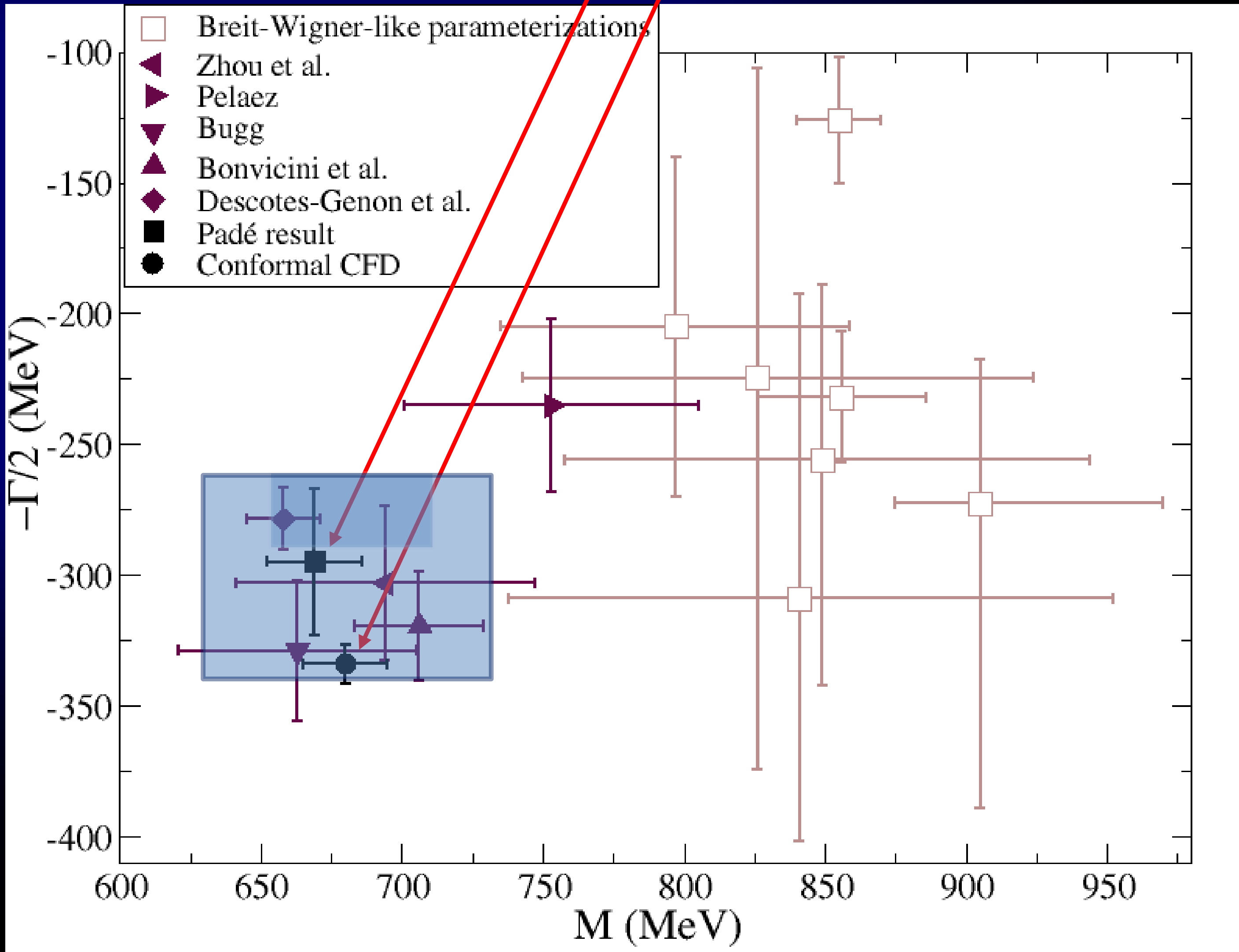
1) Extracted from our conformal CFD parameterization A.Rodas & JRP, PRD93,074025 (2016)

Fantastic analyticity properties,
but not model independent

(680±15)-i(334±7.5) MeV

2) Using Padé Sequences…
JRP, A.Rodas & J. Ruiz de Elvira. Eur. Phys. J. C (2017) 77:91

(670±18)-i(295± 28) MeV



Compare to PDG2017:
(682±29)-i(273±12) MeV

New PDG2018:
(630-730)-i(260-340) MeV
And name changed
$K_0^*(700)$
Still “Needs Confirmation”

The resonance is NO LONGER the κ nor the K_0*(800)

But Still "Needs Confirmation" !

Best analysis so far: Roy-Steiner dispersion relations

Plenty of room for improvement on parameters

Our Pade sequences

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Partial-wave πK Dispersion Relations
Need ππ→KK to rewrite left cut. Range optimized.

- As ππ→KK checks: Small inconsistencies.
- As constraints:
  ππ→KK consistent fits up to 1.5 GeV
  JRP, A.Rodas, Eur.Phys.J. C78 (2018)

g^I_J =ππ → KK partial waves. We study (I,J)=(0,0),(1,1),(0,2)

f^I_J = Kπ → Kπ partial waves. Taken from previous dispersive study

JRP, A. Rodas PRD 2016

$$g_0^0(t) = \frac{\sqrt{3}}{2} m_+ a_0^+ + \frac{t}{\pi}\int_{4m_\pi^2}^{\infty} \frac{\mathrm{Im}\, g_0^0(t')}{t'(t'-t)}dt' + \frac{t}{\pi}\sum_{\ell\geq 2}\int_{4m_\pi^2}^{\infty} \frac{dt'}{t'} G^0_{0,2\ell-2}(t,t')\mathrm{Im}\, g^0_{2\ell-2}(t') + \sum_{\ell}\int_{m_+^2}^{\infty} ds' G^+_{0,\ell}(t,s')\mathrm{Im}\, f^+_\ell(s'),$$

$$g_1^1(t) = \frac{1}{\pi}\int_{4m_\pi^2}^{\infty} \frac{\mathrm{Im}\, g_1^1(t')}{t'-t}dt' + \sum_{\ell\geq 2}\int_{4m_\pi^2}^{\infty} dt' G^1_{1,2\ell-1}(t,t')\mathrm{Im}\, g^1_{2\ell-1}(t') + \sum_{\ell}\int_{m_+^2}^{\infty} ds' G^-_{1,\ell}(t,s')\mathrm{Im}\, f^-_\ell(s'),$$

$$g_2^0(t) = \frac{t}{\pi}\int_{4m_\pi^2}^{\infty} \frac{\mathrm{Im}\, g_2^0(t')}{t'(t'-t)}dt' + \sum_{\ell\geq 2}\int_{4m_\pi^2}^{\infty} \frac{dt'}{t'} G'^0_{2,4\ell-2}(t,t')\mathrm{Im}\, g^0_{4\ell-2}(t') + \sum_{\ell}\int_{m_+^2}^{\infty} ds' G'^+_{2,\ell}(t,s')\mathrm{Im}\, f^+_\ell(s'). \quad (39)$$

$G^I_{J,J'}$(t,t') =integral kernels, depend on a parameter

Lowest # of subtractions. Odd pw decouple from even pw.

$$g_\ell^0(t) = \Delta_\ell^0(t) + \frac{t}{\pi}\int_{4m_\pi^2}^{\infty} \frac{dt'}{t'}\frac{\mathrm{Im}\, g_\ell^0(t)}{t'-t}, \qquad \ell = 0, 2,$$

$$g_1^1(t) = \Delta_1^1(t) + \frac{1}{\pi}\int_{4m_\pi^2}^{\infty} dt' \frac{\mathrm{Im}\, g_1^1(t)}{t'-t}, \quad (40)$$

Δ(t) depend on higher waves or on Kπ→Kπ.

Integrals from 2π threshold !

Solve in descending J order

We have used models for higher waves, but give very small contributions

For unphysical region below KK threshold, we used Omnés function

$$\Omega^I_\ell(t) = \exp\left(\frac{t}{\pi}\int_{4m_\pi^2}^{t_m}\frac{\phi^I_\ell(t')dt'}{t'(t'-t)}\right),$$

$$\Omega^I_\ell(t) \equiv \Omega^I_{l,R}(t)e^{i\phi^I_\ell(t)\theta(t-4m_\pi^2)\theta(t_m-t)},$$

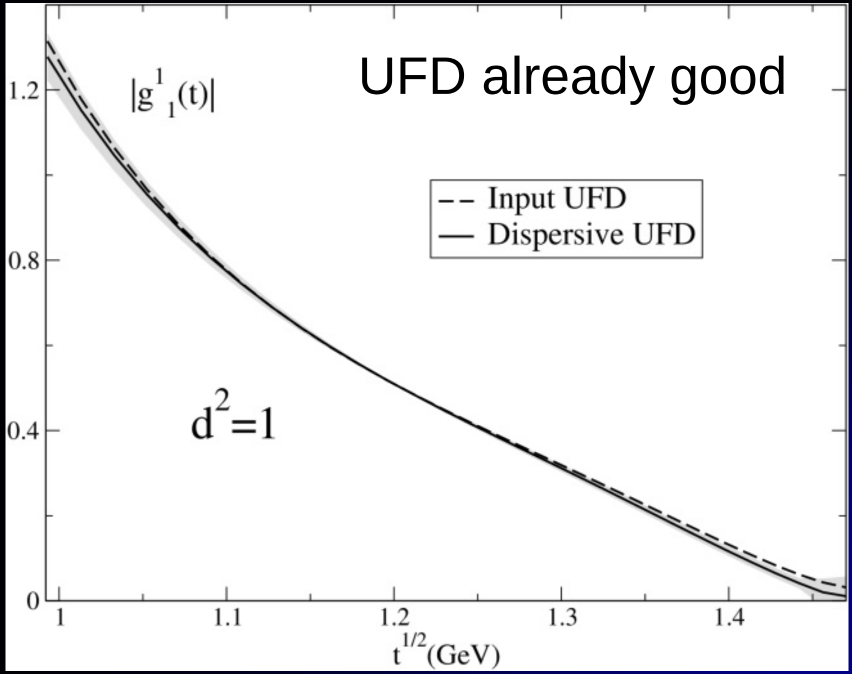
This is the form of our HDR: Roy-Steiner+Omnés formalism

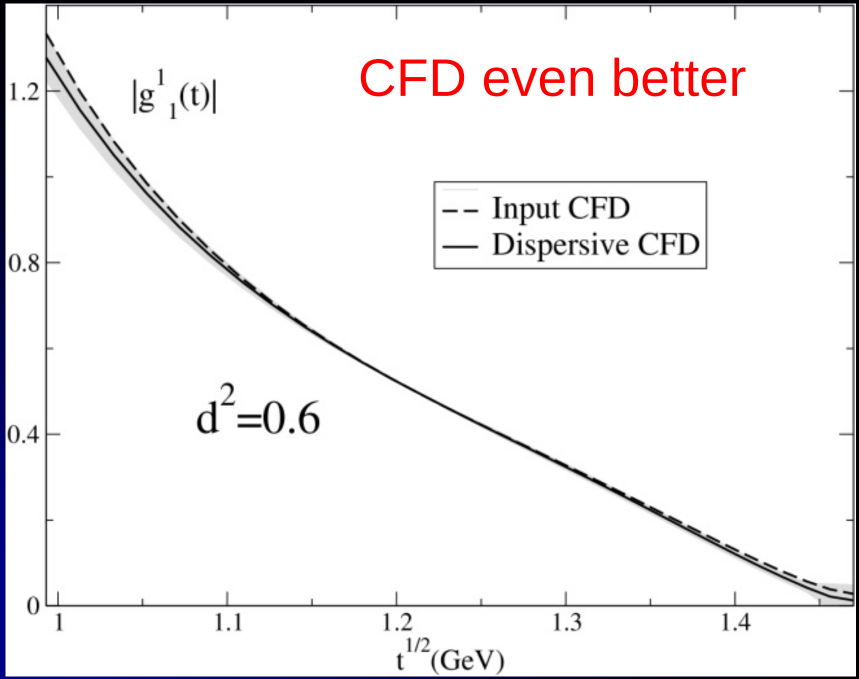
$$\begin{aligned}
g^0_0(t) &= \Delta^0_0(t) + \frac{t\Omega^0_0(t)}{t_m - t}\left[\alpha + \frac{t}{\pi}\int_{4m_\pi^2}^{t_m} dt' \frac{(t_m - t')\Delta^0_0(t')\sin\phi^0_0(t')}{\Omega^0_{0,R}(t')t'^2(t'-t)} + \frac{t}{\pi}\int_{t_m}^{\infty} dt' \frac{(t_m - t')|g^0_0(t')|\sin\phi^0_0(t')}{\Omega^0_{0,R}(t')t'^2(t'-t)}\right] \\
g^1_1(t) &= \Delta^1_1(t) + \Omega^1_1(t)\left[\frac{1}{\pi}\int_{4m_\pi^2}^{t_m} dt' \frac{\Delta^1_1(t')\sin\phi^1_1(t')}{\Omega^1_{1,R}(t')(t'-t)} + \frac{1}{\pi}\int_{t_m}^{\infty} dt' \frac{|g^1_1(t')|\sin\phi^1_1(t')}{\Omega^1_{1,R}(t')(t'-t)}\right], \\
g^0_2(t) &= \Delta^0_2(t) + t\Omega^0_2(t)\left[\frac{1}{\pi}\int_{4m_\pi^2}^{t_m} dt' \frac{\Delta^0_2(t')\sin\phi^0_2(t')}{\Omega^0_{2,R}(t')t'(t'-t)} + \frac{1}{\pi}\int_{t_m}^{\infty} dt' \frac{|g^0_2(t')|\sin\phi^0_2(t')}{\Omega^0_{2,R}(t')t'(t'-t)}\right].
\end{aligned}$$

We can now check how well these HDR are satisfied

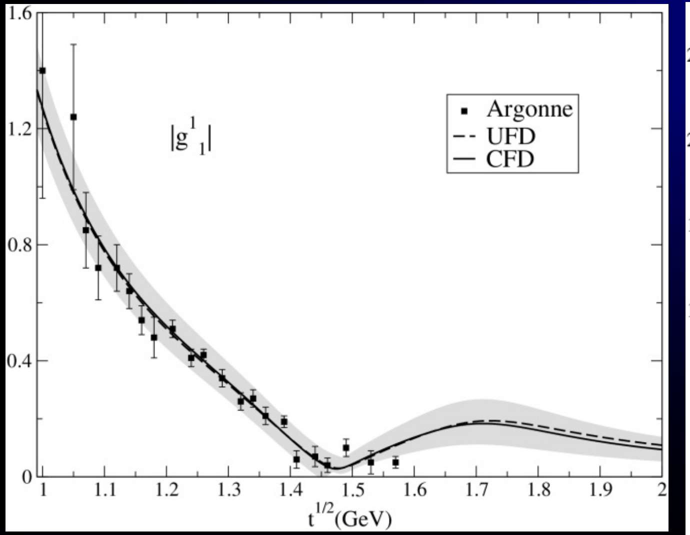
ππ→KK Hiperbolic Dispersion Relations   I=1,J=1, UFD vs.CFD

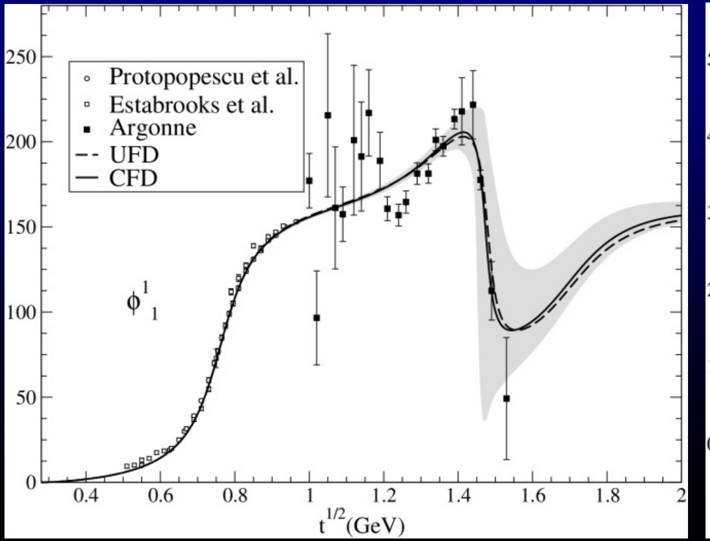
JRP, A.Rodas, Eur.Phys.J. C78 (2018)

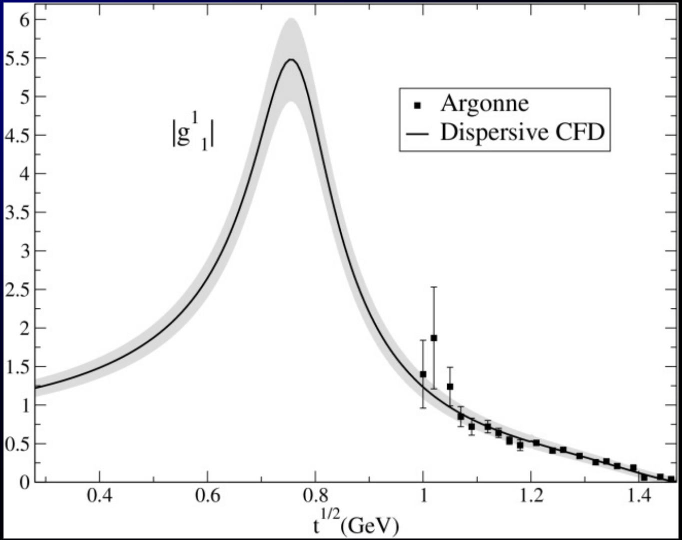




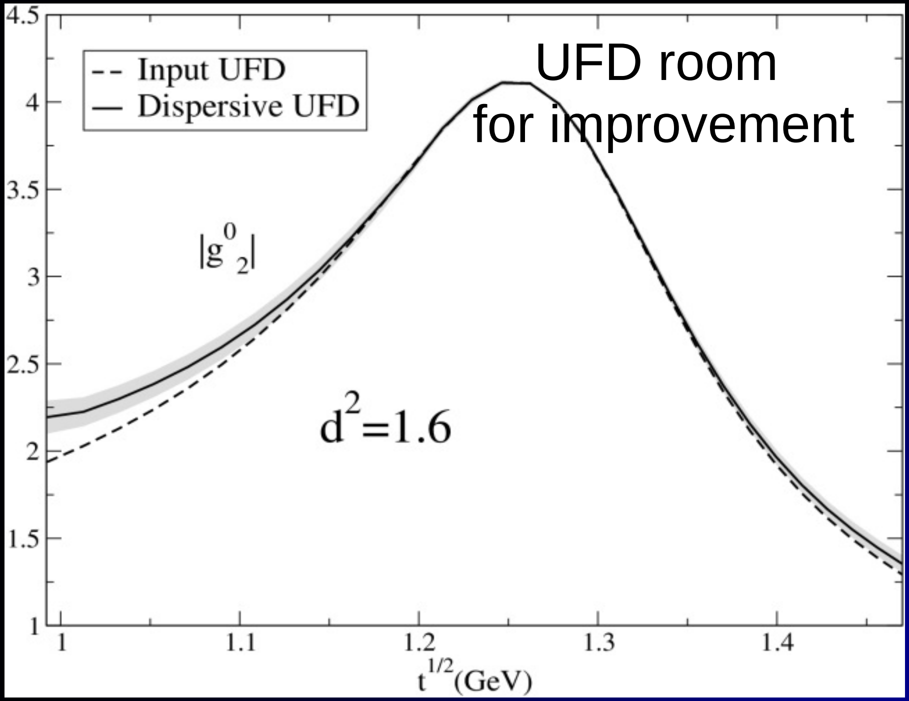
Requires almost imperceptible change from UFD to CFD

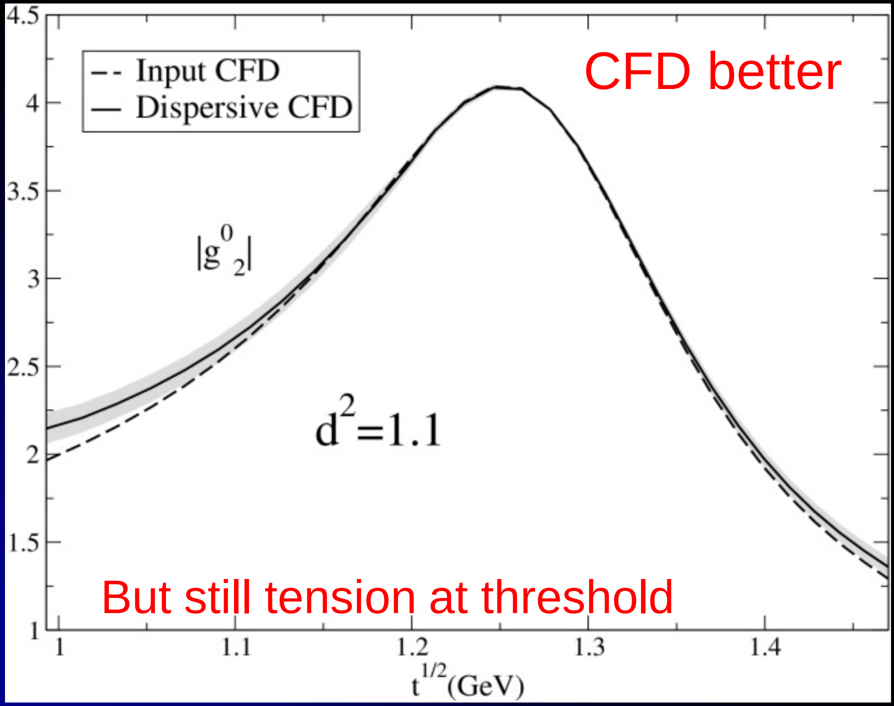




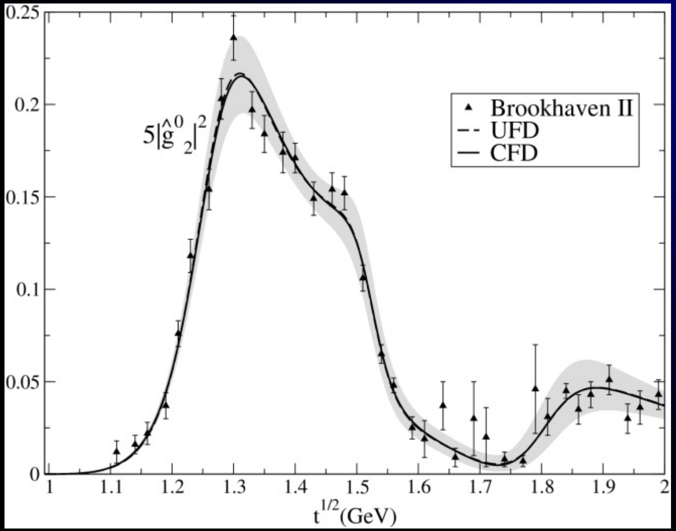


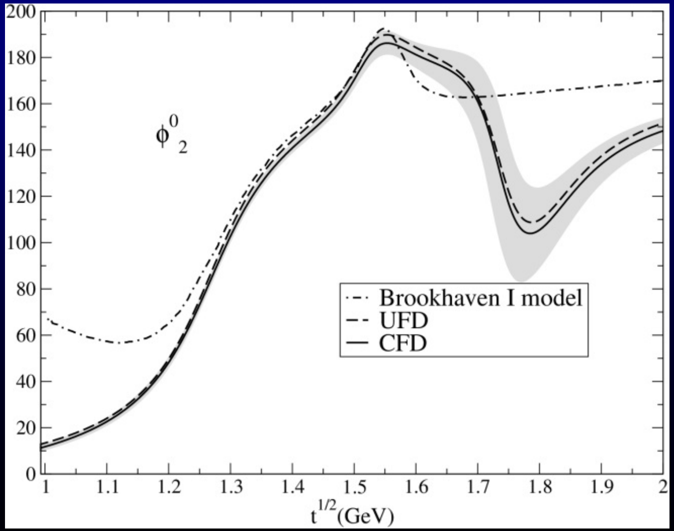
 JRP, A.Rodas, Eur.Phys.J. C78 (2018)





Very small change from UFD to CFD. Only significant at threshold and high energies

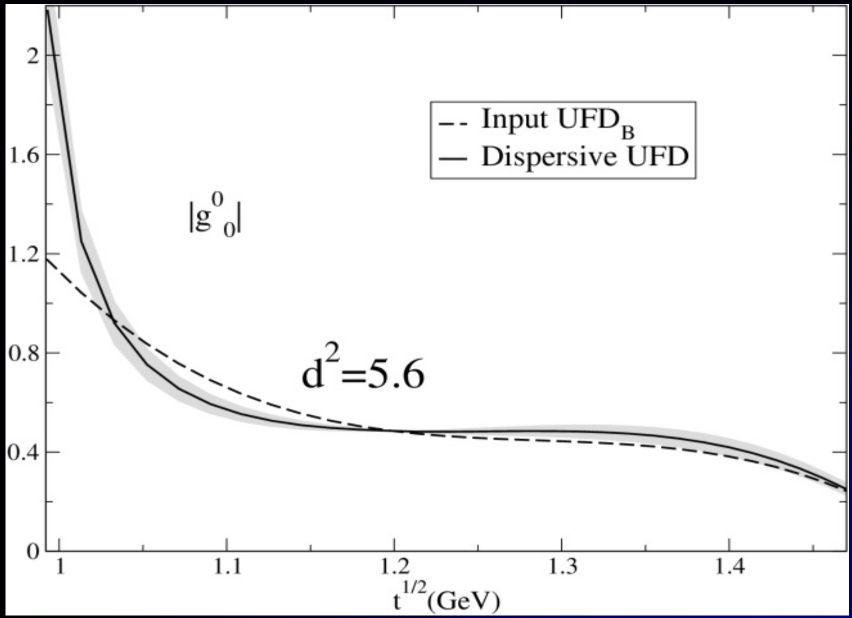


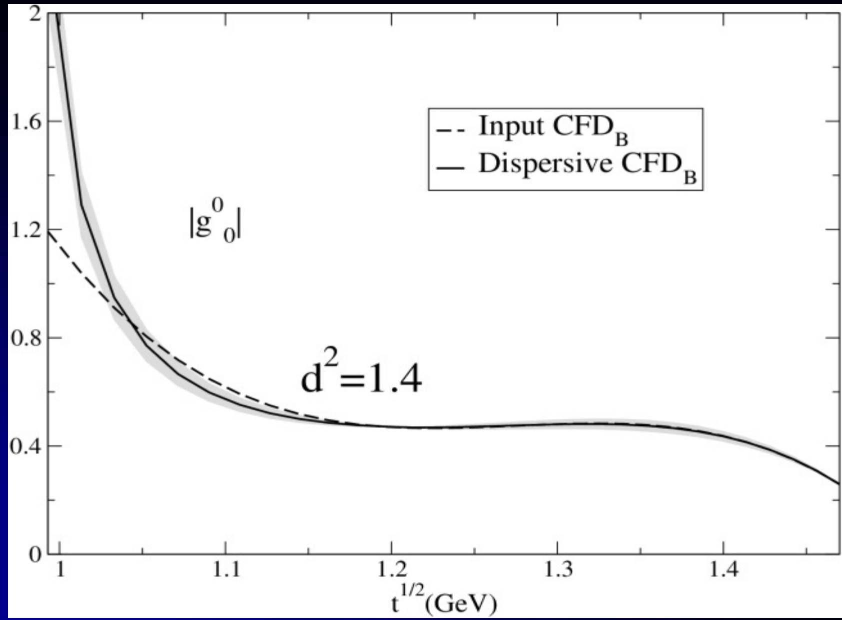


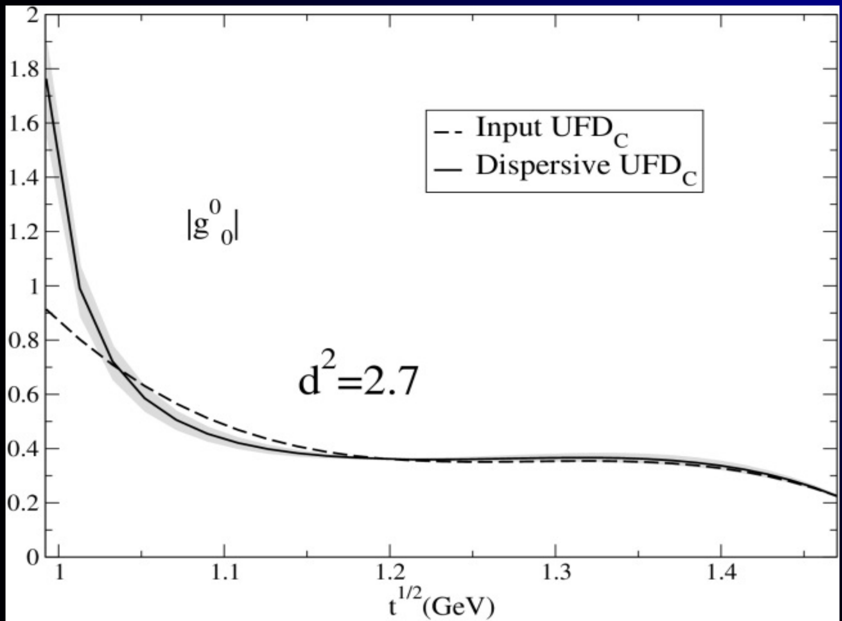
Other parameterizations (BW…), worse.

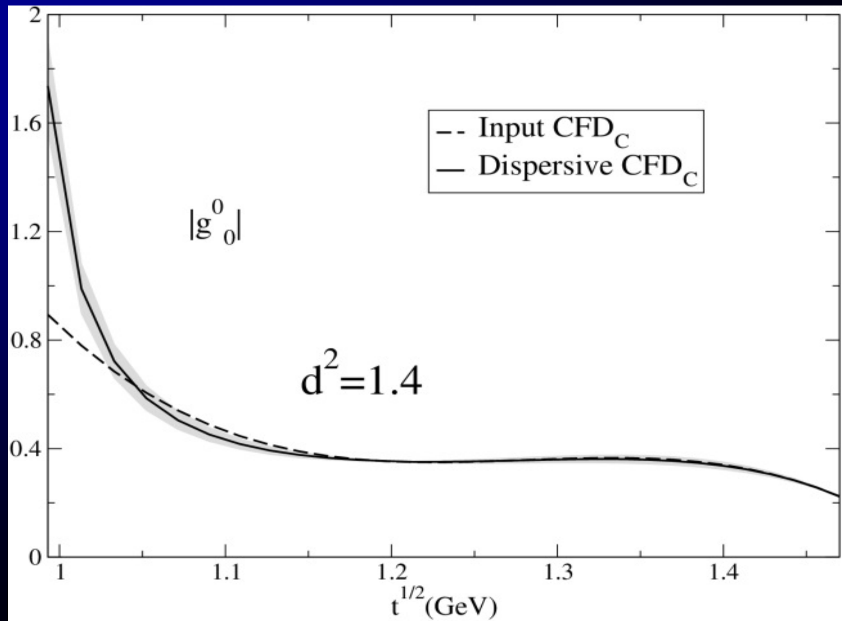
Two possible sets of data

We use I=0,J=2 CFD as input.



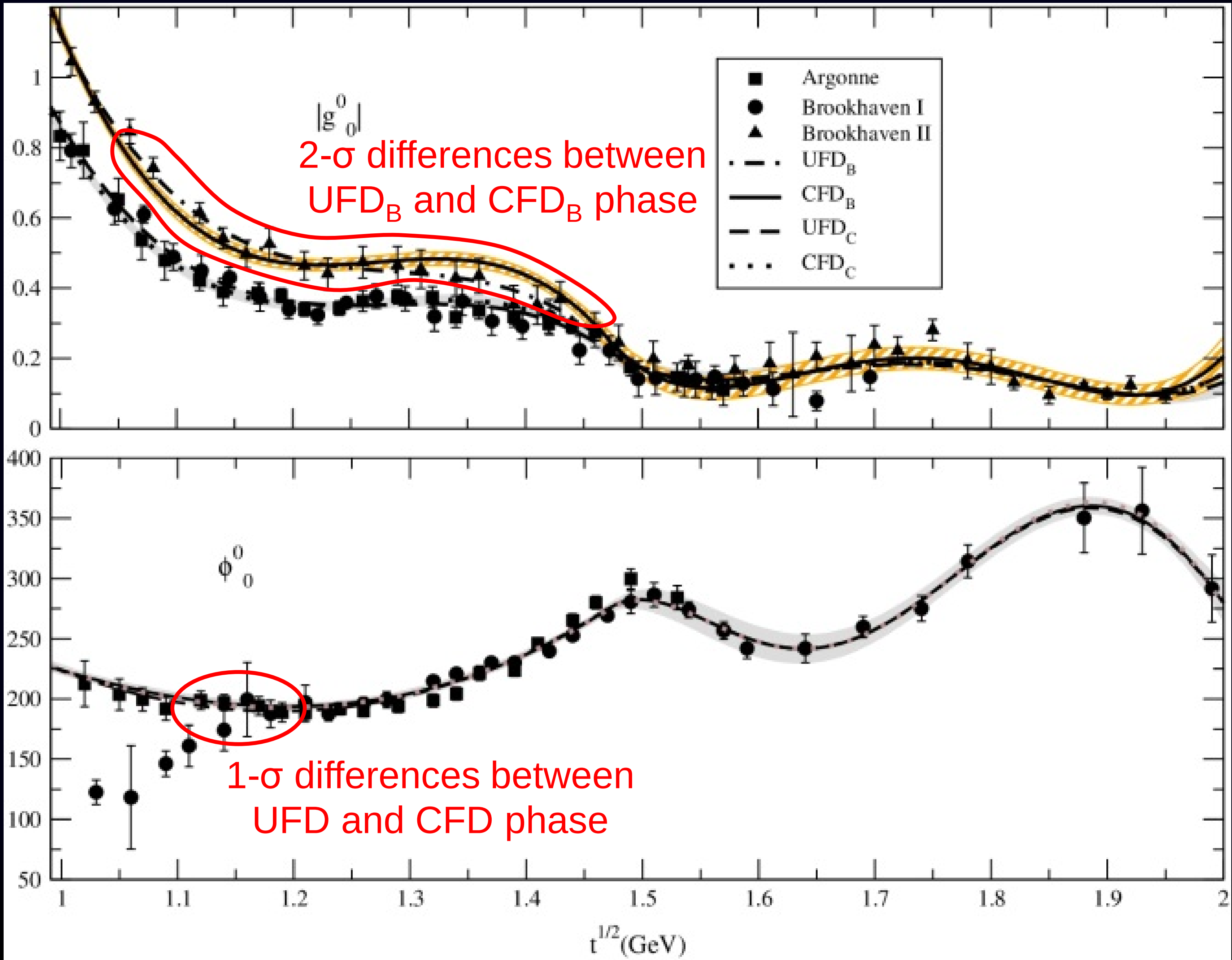






Remarkable improvement from UFD to CFD, except at threshold.
Both data sets equally acceptable now.

Some 2-σ level differences between UFD_B and CFD_B between 1.05 and 1.45 GeV
CFD_C consistent within 1-σ band of UFD_C



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  $\kappa/K_0^*(700)$ out of reach
- From Hyperbolic DR:
  ππ→KK influence important.
  JRP, A.Rodas, in progress. PRELIMINARY results shown here

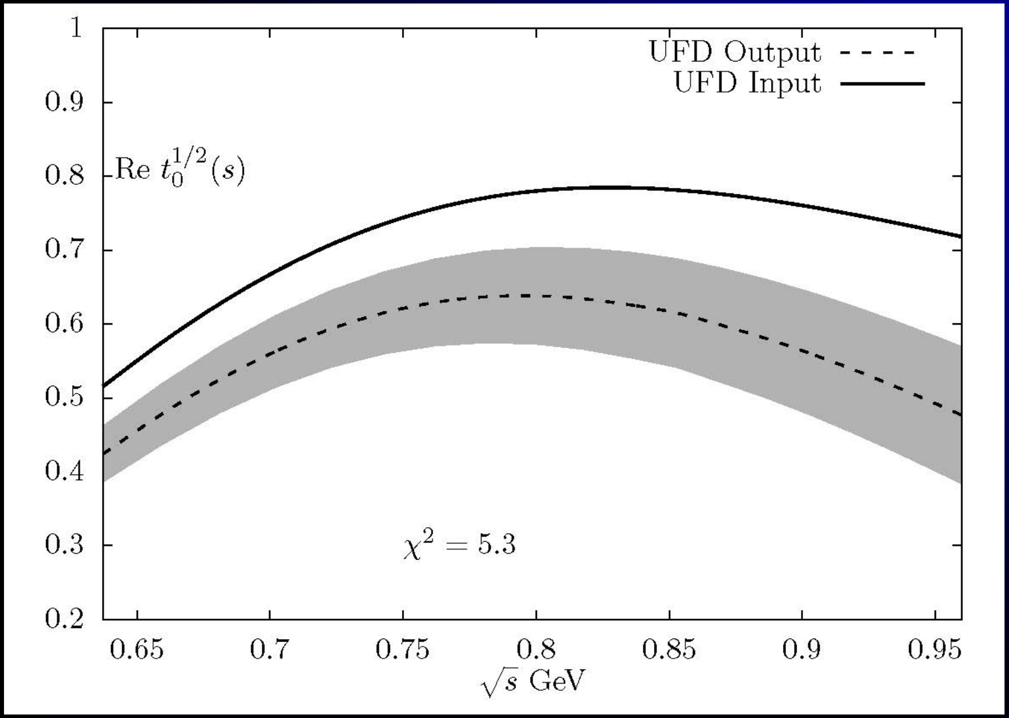
- As πK Checks: Large inconsistencies.
- **ALL DR TOGETHER** as Constraints:
  πK consistent fits up to 1.1 GeV

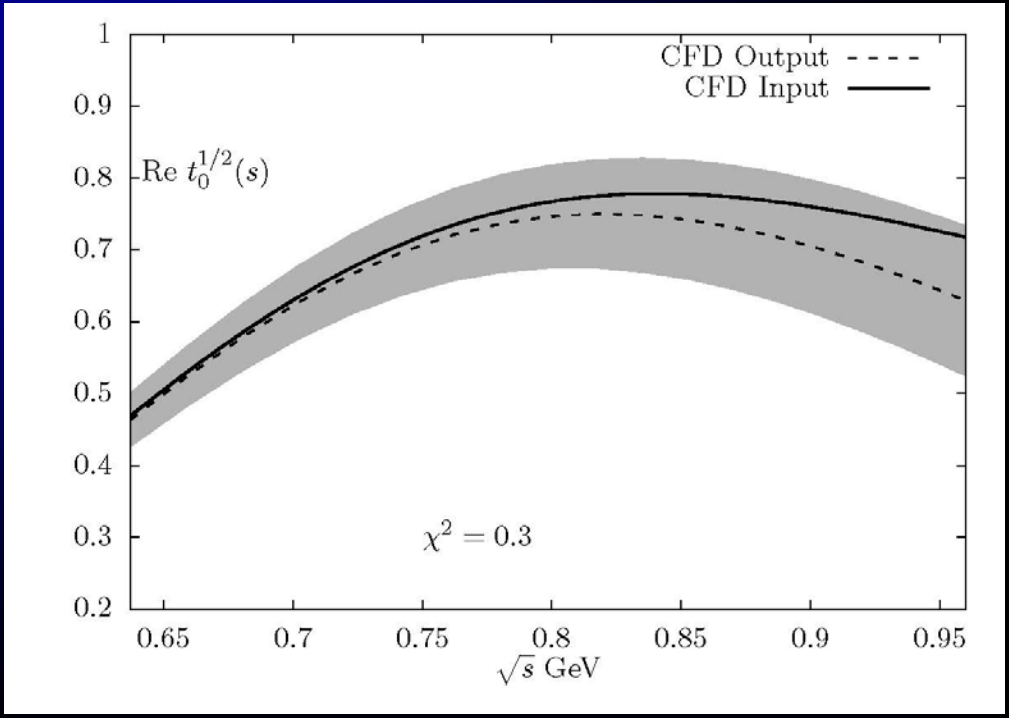
πK Hiperbolic Dispersion Relations I=1/2, J=0

Preliminary!!

LARGE inconsistencies of unconstrained fits with the minimal number of subtractions (shown here)
Fairly consistent with one more subtraction for F^-

Consistent within uncertainties
if we use the DR as constraints



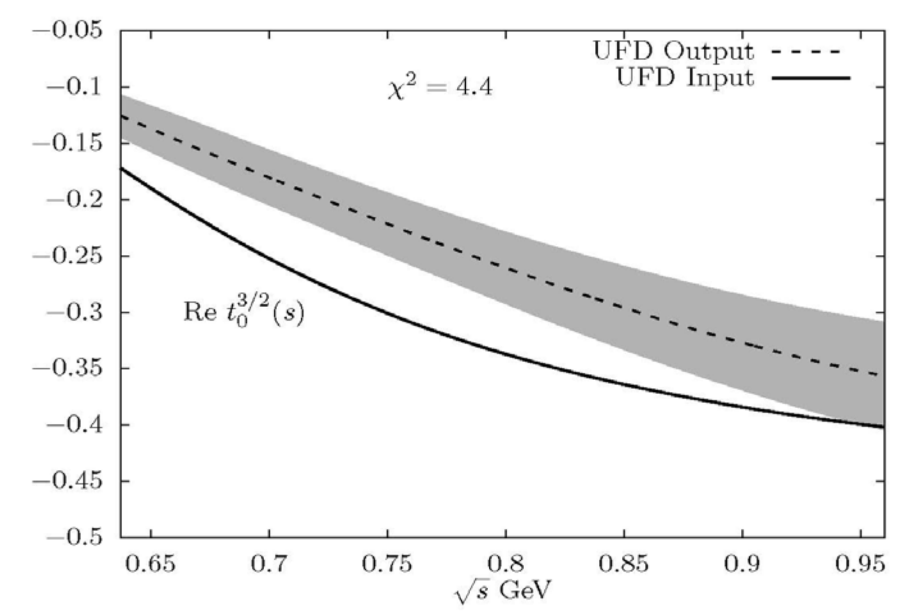


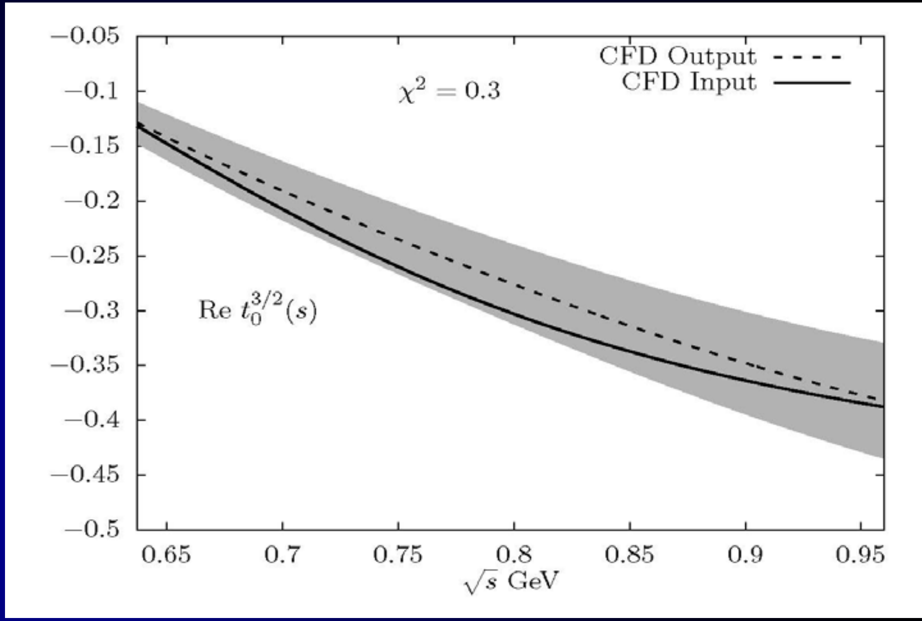
πK Hiperbolic Dispersion Relations I=3/2, J=0 and I=1/2, J=0

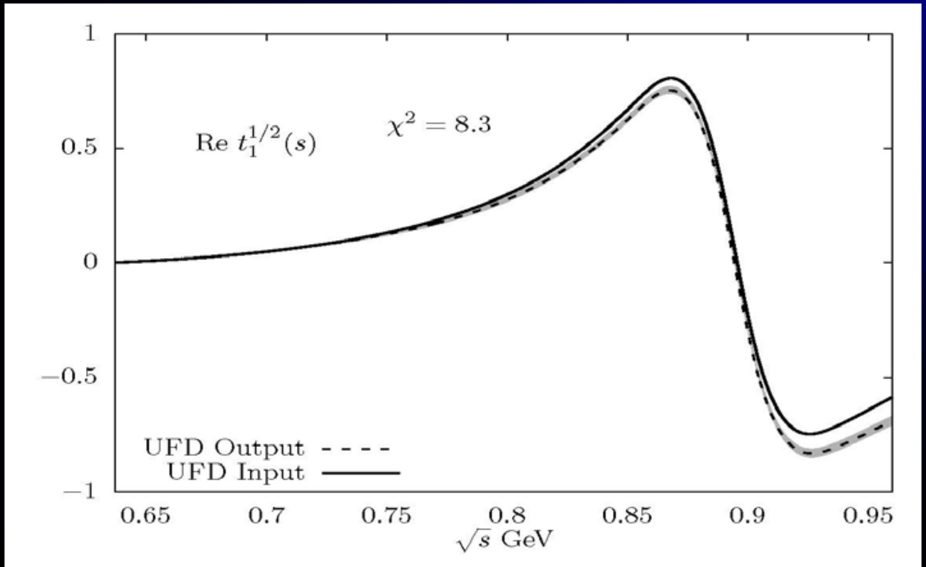
Preliminary!!

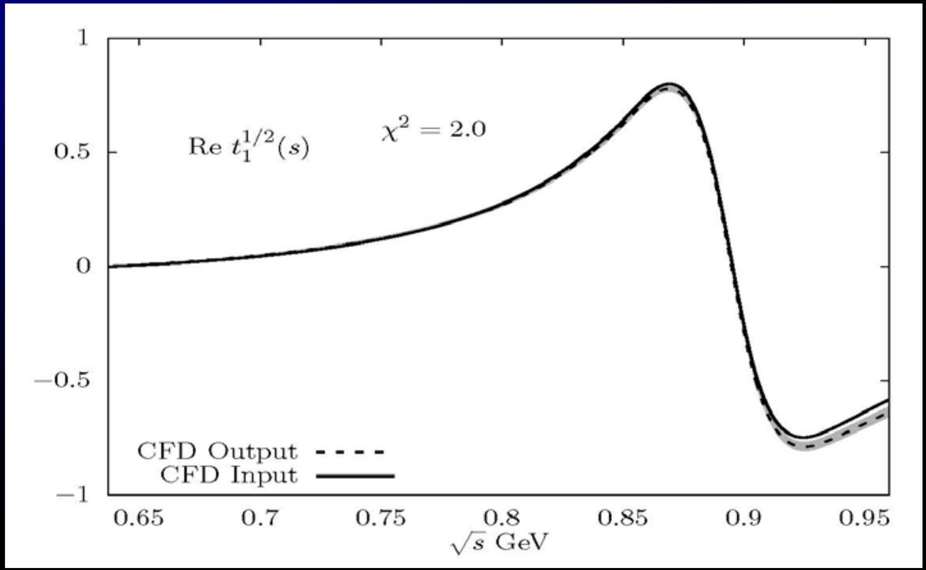
SIZABLE inconsistencies of unconstrained fits with the minimal number of subtractions (shown here). Fairly consistent with one more subtraction for F^-

Made consistent within uncertainties when we use the DR as constraints





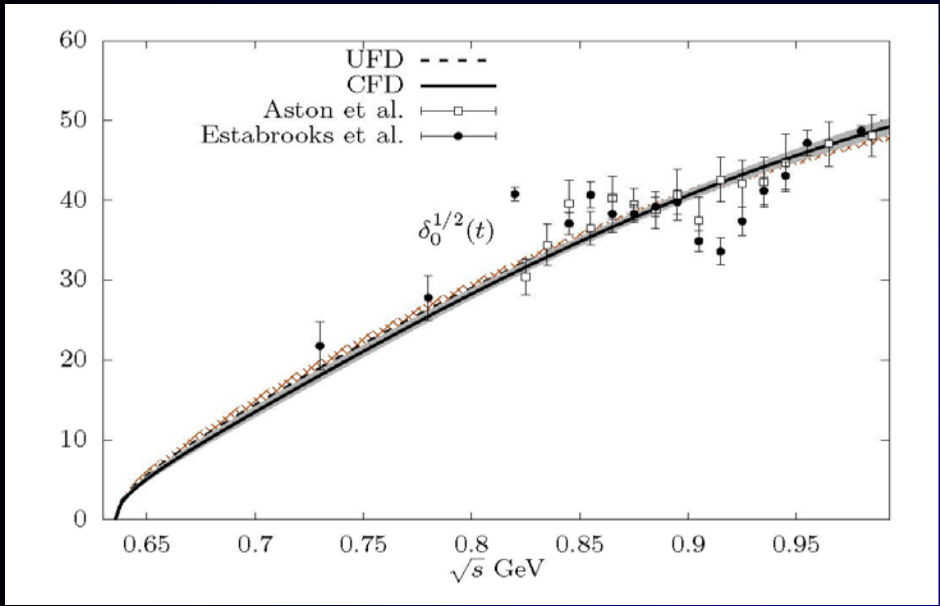


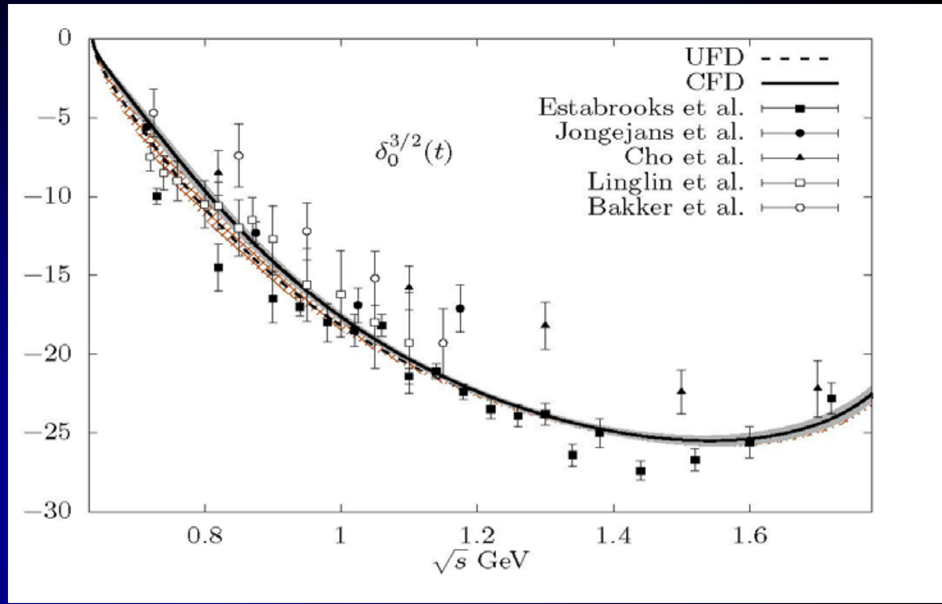


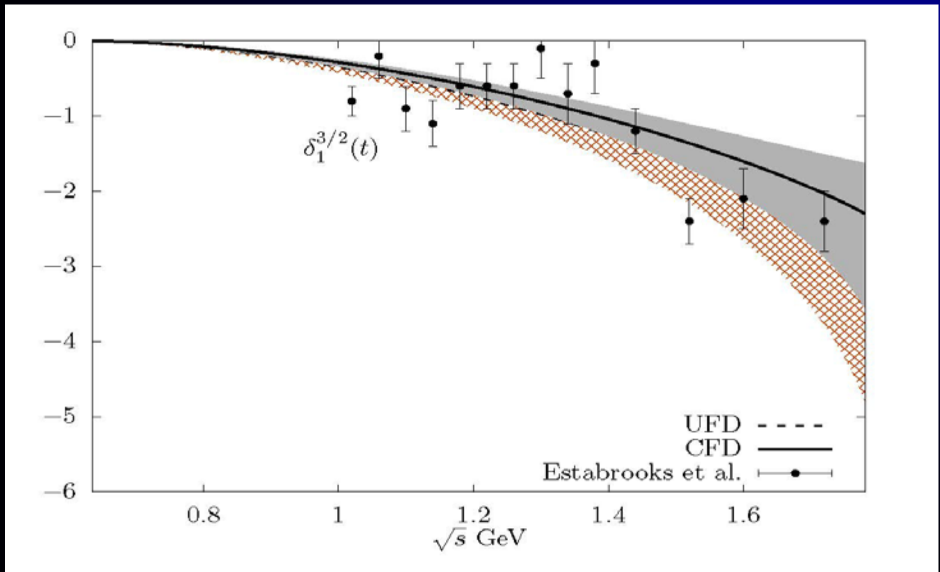
πK CFD vs. UFD

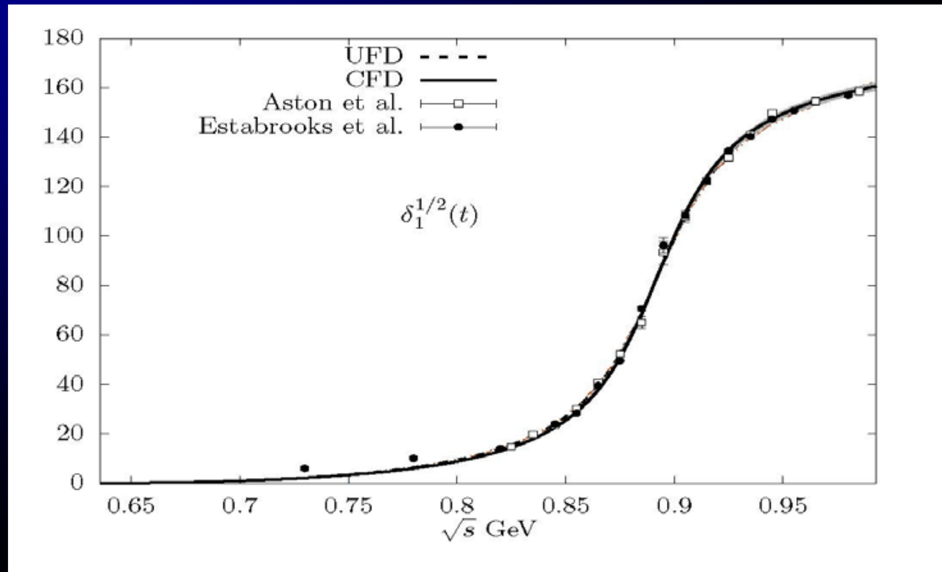
Preliminary!!

Constrained parameterizations suffer minor changes but still describe πK data fairly well. Here we compare the unconstrained fits (UFD) versus the constrained ones (CFD)









The “unphysical” rho peak in ππ→KK grows by 10% from UFD to CFD

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- From Hyperbolic DR:
  ππ→KK influence important.

- As πK Checks: Large inconsistencies.
- **ALL DR TOGETHER** as Constraints:
  πK consistent fits up to 1.1 GeV
- **Rigorous $\kappa/K_0^*(700)$ pole**

JRP, A.Rodas, in progress.
PRELIMINARY results shown here

Dispersive pole analysis from constrained fit to data JRP, A. Rodas, in preparation

Preliminary!!

Recall Roy-Steiner SOLUTION from Paris group (658±13)-i(278.5±12) MeV

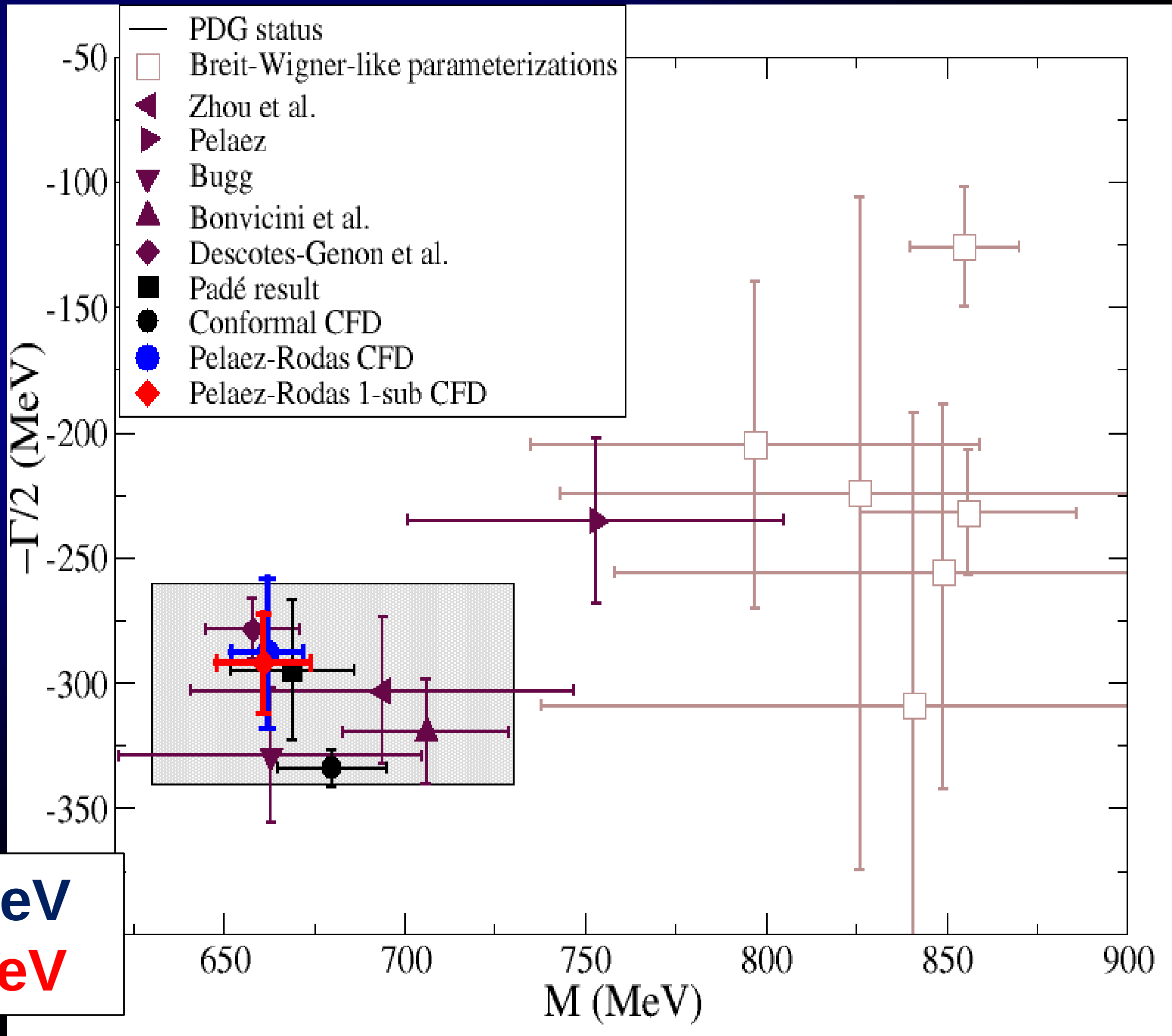
Decotes-Genon-Moussallam 2006

Now we have:

- Constrained **FIT TO DATA** (not solution but fit)
- Improved P-wave (consistent with data)
- Realistic ππ→KK uncertainties (none before)
- Improved Pomeron

- Constrained ππ→KK input with DR
- FDR up to 1.6 GeV
- Fixed-t Roy-Steiner Eqs.
- Hyperbolic Roy Steiner Eqs.
  both in real axis (not before)
  and complex plane

- Both one and no-subtraction
  for F- HDR (only the subtracted one before)



No sub: (662± 9)-i(288±31) MeV

1 sub: (661±13)-i(293±20) MeV

Summary

- The ππ → ππ, πK →πK and ππ →KK data do not satisfy well basic dispersive constraints
- Using dispersion relations as constraints we provide, **model independent, precise, consistent, simple and easy to implement data** parameterizations.
- NEW ππ → ππ analytic expressions up to 2 GeV, consistent with Dipersion theory up to 1.4 GeV
- Simple analytic methods of complex analysis can then reduce the model dependence in resonance parameter determinations.
- This settled the $f_0(500)/\sigma$ parameters debate.
- We are implementing partial-wave dispersion relations whose applicability range reaches the $K^*_0(700)/\kappa$ pole. Our preliminary results confirm previous studies. We believe this resonance should be considered "well-established", completing the nonet of lightest scalars.