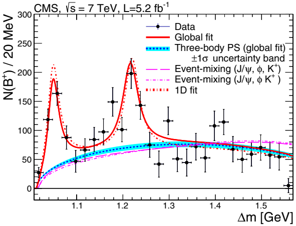
Modern hadron spectroscopy on the lattice: From bound state masses to scattering amplitudes

Daniel Mohler

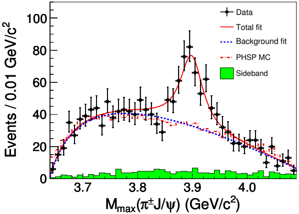
Darmstadt,
May 8th, 2019

15 years after the $X(3872)$, $D_{s0}^*(2317)$: Many new puzzles

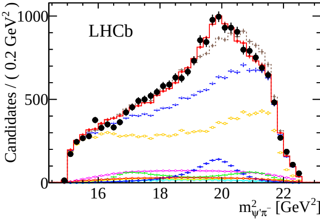
$Y(4140)$: CDF, CMS



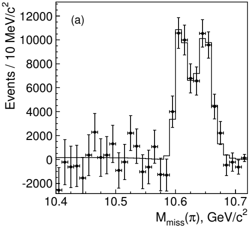
$Z_c(3900)^{\pm}$: BESIII, Belle, data from Cleo



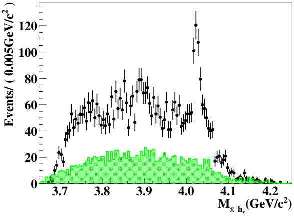
$Z(4430)^{\pm}$: Belle, LHCb



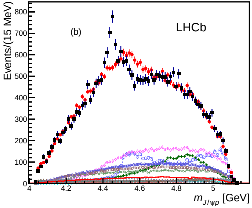
$Z_b(10610)^+, Z_b(10650)^+$: Belle



$Z_c(4020)^{\pm}$: BESIII



$P_c(4450), P_c(4380)$: LHCb



My method of choice: Lattice QCD

Regularization of QCD by a 4-d Euclidean space-time lattice. (Kenneth Wilson 1974)
Provides a calculational method for QCD

Euclidean correlator of two Hilbert-space operators $\hat{O}_1$ and $\hat{O}_2$.

$$\left\langle \hat{O}_2(t)\hat{O}_1(0) \right\rangle = \sum_n e^{-t\Delta E_n} \langle 0|\hat{O}_2|n\rangle\langle n|\hat{O}_1|0\rangle$$
$$= \frac{1}{Z}\int \mathcal{D}[\psi,\bar{\psi},U]e^{-S_E} O_2[\psi,\bar{\psi},U] O_1[\psi,\bar{\psi},U]$$

- Path integral over the Euclidean action $S_{E,QCD}[\psi,\bar{\psi},U]$; (a sum over quantum fluctuations)
- Can be evaluated with *Markov Chain Monte Carlo* (using methods well established in statistical physics)

Motivation vs. lattice reality

Goal: Learn about the nature of exotic hadrons with heavy quarks

The purpose of computing is insight, not numbers

– Richard Hamming

- Lattice calculation faces some obstacles
  - Need to take the *continuum limit*: $a(g,m) \to 0$
  - taking the *infinite volume limit*: $L \to \infty$
  - Need to calculate at (or extrapolate to) the physical pion mass
- So far largely: *exploratory* results for the excited state spectrum (often single pion mass/ lattice spacing)
  - Should be compared only qualitatively to experiment
  - Provide an outlook on future Lattice QCD results
  - Provides very limited information on structure/nature of states

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  - Want to exploit (power law) finite volume effects (keeping exponential effects small)
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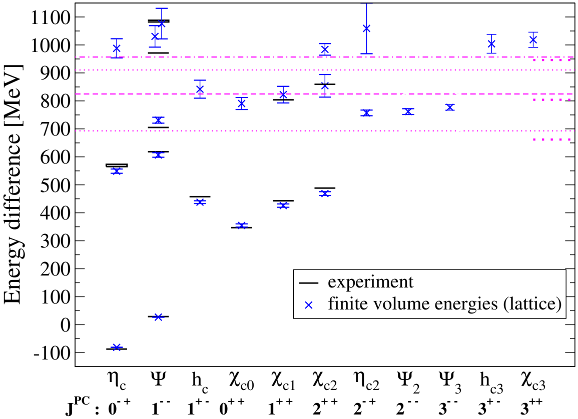
Example for a qualitative spectrum at $m_\pi = 266$ MeV

Data from Mohler, Prelovsek, Woloshyn, PRD 87 034501 (2013)



- Pattern of the spectrum looks quark-model like

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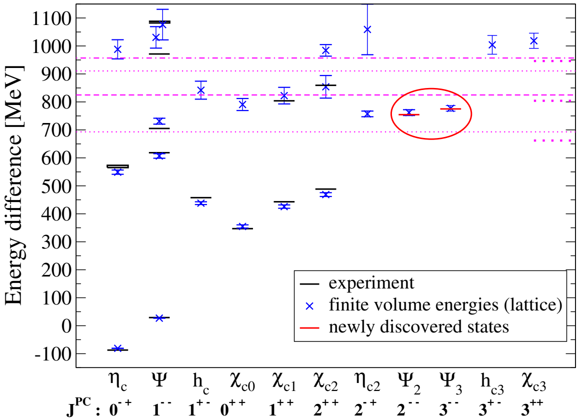

- Predicted new states surprisingly well (accidental)

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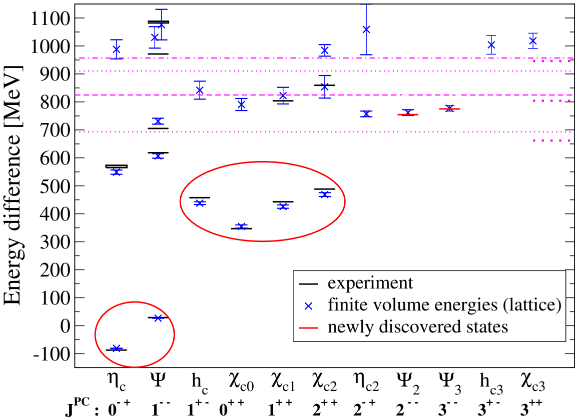

- Well-established ground states show significant deviations

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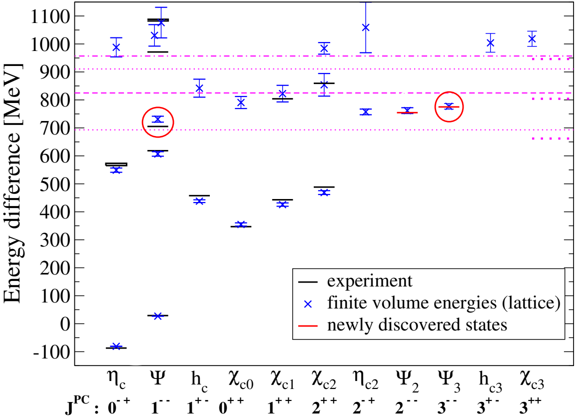

- What about low-lying resonances?

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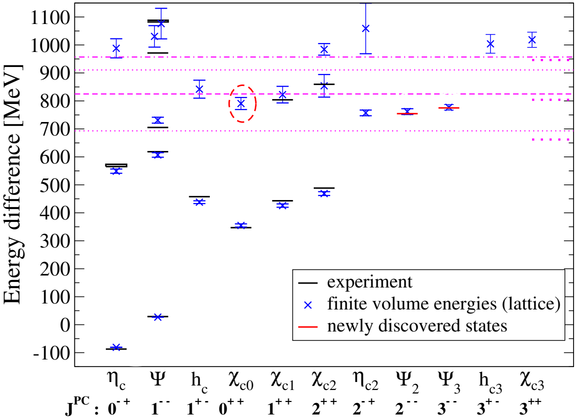


- What about more interesting states?

Outline

Determining the finite-volume spectrum

Observables from Euclidean space correlation functions

$$\left\langle \hat{O}_2(t)\hat{O}_1(0)\right\rangle_T \propto \sum_n e^{-tE_n} < 0|\hat{O}_2|n >< n|\hat{O}_1|0 >$$

Need: *Interpolating field* creating states with desired quantum numbers.

$$O_\pi = \bar{u}\gamma_5 d \qquad \text{Meson with} \quad IJ^{PC} = 10^{-+}$$

$$O_N = \epsilon_{abc}\,\Gamma_1\,u_a\left(u_b^T\,\Gamma_2\,d_c - d_b^T\,\Gamma_2\,u_c\right) \qquad J = \frac{1}{2} \quad \text{nucleon}$$

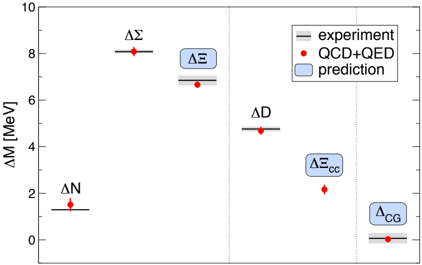
In practice: Use a matrix of correlation functions:

$$C(t)_{ij} = \sum_n \mathrm{e}^{-tE_n}\,\langle 0|O_i|n\rangle \left\langle n|O_j^\dagger|0\right\rangle$$

- Need a diverse basis to get the full energy spectrum
- Correlator matrix gives access to excited states

Recent progress in Lattice QCD

- Dynamical simulations with 2+1(+1) flavors of sea quarks
- Simulations at physical pion (light-quark) masses
- Isospin splitting and QCD+QED simulations
- Improved heavy quark actions for charm
- Finite-volume methods for determining scattering amplitudes



BMW Collaboration, Borsanyi et al. Science 347 1452 (2015)

Outline

Low-lying charmonium: A precision benchmark

Fermilab Lattice and MILC Collaborations, PR**D** 99, 034509 (2019)

- Well understood from models and well determined in experiment
- Spin-dependent mass splittings extremely sensitive to the charm-quark mass and heavy-quark discretization $\rightarrow$ good benchmark

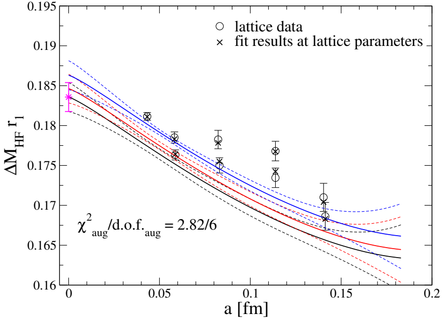
meson	mass	width
η_c	2983.9(5)	32.0(8) MeV
J/Ψ	3096.900(6)	92.9(2.8) keV
χ_{c0}	3414.71(30)	10.8(6) MeV
χ_{c1}	3510.67(5)	0.84(4) MeV
χ_{c2}	3556.17(7)	1.97(9) MeV
h_c	3525.38(11)	0.7(4) MeV
$\eta_c(2S)$	3637.6(1.2)	$11.3^{(+3.2)}_{(-2.9)}$ MeV
$\Psi(2S)$	3686.097(25)	294(8) keV

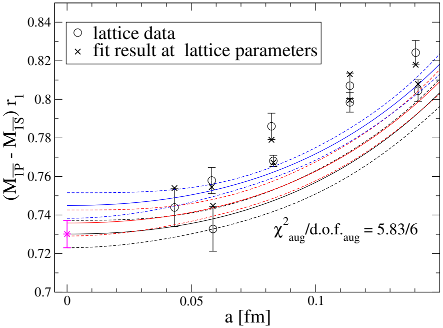
FNAL-MILC 1S hyperfine splittings and 1P-1S splittings

Fermilab Lattice and MILC Collaborations, PR**D** 99, 034509 (2019)

$$\Delta M_{\text{HF}} = M_{J/\psi} - M_{\eta_c}$$

$$\Delta M_{\text{1P-1S}} = M_{\overline{1P}} - M_{\overline{1S}}$$



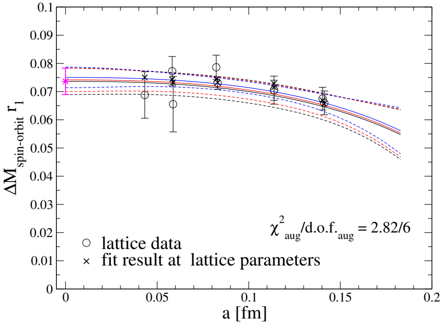


Mass difference	This analysis [MeV]	Experiment [MeV]
1S hyperfine	$116.2 \pm 1.1 \pm 3.3^{-1.5}_{-4.0}$	113.0 ± 0.5
1P1S	$462.2 \pm 4.5 \pm 3.3$	456.64 ± 0.14

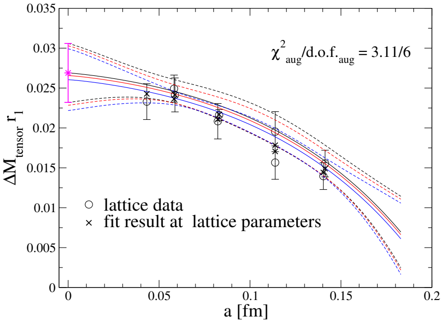
FNAL-MILC P-wave spin-orbit and tensor splittings

Fermilab Lattice and MILC Collaborations, PR**D** 99, 034509 (2019)

$$\Delta M_{\mathrm{Spin-Orbit}} = (5M_{\chi_{c2}} - 3M_{\chi_{c1}} - 2M_{\chi_{c0}})/9$$



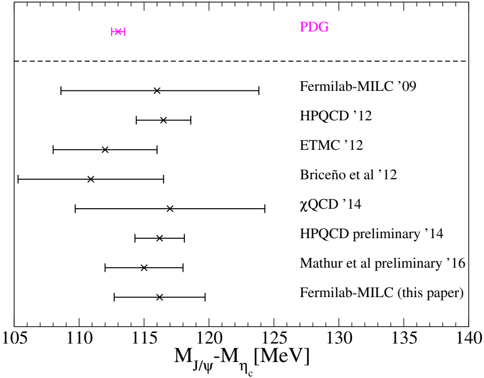
$$\Delta M_{\mathrm{Tensor}} = (3M_{\chi_{c1}} - M_{\chi_{c2}} - 2M_{\chi_{c0}})/9$$



Mass difference	This analysis [MeV]	Experiment [MeV]
1P spin-orbit	$46.6 \pm 3.0 \pm 0.9$	46.60 ± 0.08
1P tensor	$17.0 \pm 2.3 \pm 1.6$	16.27 ± 0.07

A comparison of hyperfine splittings

Fermilab Lattice and MILC Collaborations, PR**D** 99, 034509 (2019)



- All results at physical quark masses and in the continuum limit
- Lattice numbers exclude annihilation effects
- Estimate from data expects a shift of -1.5..-4.5 MeV

Levkova and DeTar, PRD 83 074504, 2011

Outline

1. Introduction
   - Motivation
   - Lattice QCD basics
2. Precision spectroscopy of charmonia below $\bar{D}D$
   - Charmonium spectra from single-hadron interpolators
3. A simple resonance example: The ρ in $I = 1$ $\pi\pi$-scattering
   - Spectroscopy and timelike pion form factor
4. Selected results for heavy mesons
   - Postive parity heavy-strange hadrons (D_s and B_s)
   - $\Psi(3770)$ and $X(3842)$ in $\bar{D}D$ scattering
   - Some comments on the χ'_{c0} / $X(3915)$
5. Outlook

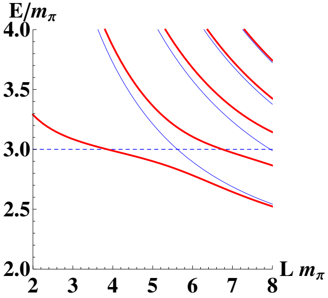
Progress from an old idea: Lüscher's finite-volume method

M. Lüscher Commun. Math. Phys. 105 (1986) 153;
Nucl. Phys. B 354 (1991) 531; Nucl. Phys. B 364 (1991) 237.

Basic observation: Finite volume, multi-particle energies are shifted with regard to the free energy levels due to the interaction

$$E = E(p_1) + E(p_2) + \Delta_E$$

- Energy shifts encode scattering amplitude(s)
- Original method: Elastic scattering in the rest-frame in multiple spatial volumes L^3
- Coupled 2-hadron channels well understood
- $2 \leftrightarrow 1$ and $2 \leftrightarrow 2$ transitions well understood (example $\pi\pi \rightarrow \pi\gamma^*$)
- significant progress for 3-particle scattering



For review see Briceno, Dudek, Young, Rev.Mod.Phys. 90, 025001 (2018)

The ρ meson: basis used

$\rho(770)$ [h] $I^G(J^{PC}) = 1^+(1^{--})$

Mass $m = 775.26 \pm 0.25$ MeV
Full width $\Gamma = 149.1 \pm 0.8$ MeV

$\rho(770)$ DECAY MODES	Fraction (Γ_i/Γ)
$\pi\pi$	$\sim$ 100

For $2m_\pi \le m_\rho \le 4m_\pi$ the original Lüscher formalism is applicable

Correlation matrix built from both quark-antiquark ρ and $\pi\pi$ interpolators:

$$C(t) = \begin{pmatrix} \langle \rho(t)\rho(0)^\dagger \rangle & \langle \rho(t)(\pi\pi)(0)^\dagger \rangle \\ \langle (\pi\pi)(t)\rho(0)^\dagger \rangle & \langle (\pi\pi)(t)(\pi\pi)(0)^\dagger \rangle \end{pmatrix}$$

Where we use ρ^0 and $\pi^+\pi^-$ type interpolators:

$$\rho^0(P,t) \propto \sum_{\mathbf{x}} \mathrm{e}^{-i\mathbf{P}\cdot\mathbf{x}} \left(\bar{u}\mathbf{a}\cdot\gamma u - \bar{d}\mathbf{a}\cdot\gamma d\right)(\mathbf{x},t)$$

$$(\pi\pi)(t) = \pi^+(\mathbf{p_1},t)\pi^-(\mathbf{p_2},t) - \pi^-(\mathbf{p_1},t)\pi^+(\mathbf{p_2},t)$$

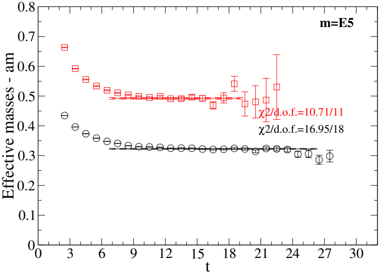
$$\pi^+(p,t) \propto \sum_{\mathbf{x}} \mathrm{e}^{-i\mathbf{p}\cdot\mathbf{x}} \bar{d}\gamma_5 u(\mathbf{x},t)$$

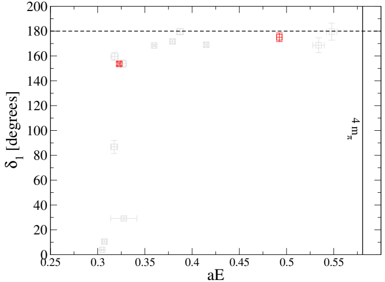
A non-exotic example: The ρ meson

- Lüscher quantization condition

$$\delta_1(k) + \phi\left(\frac{L}{2\pi}k\right) = n\pi \quad \text{with} \quad E_{cm}(k) = 2\sqrt{k^2 + m_\pi^2}$$

- In this simple case of elastic scattering:
  one phase-shift point for each energy level





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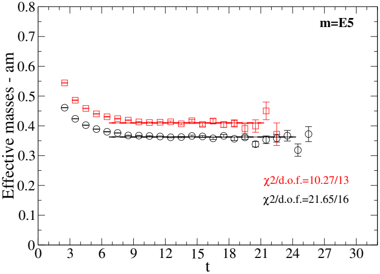

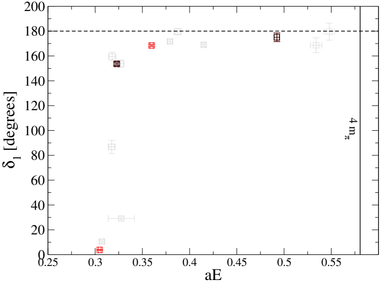

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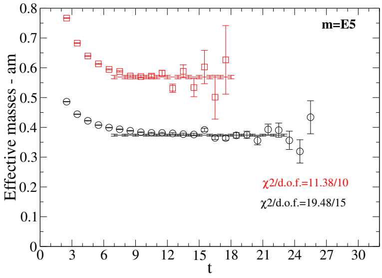

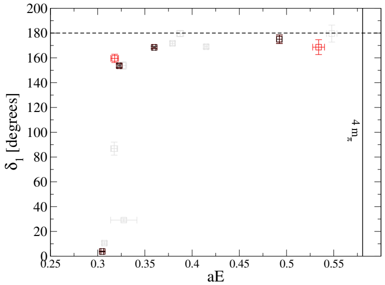

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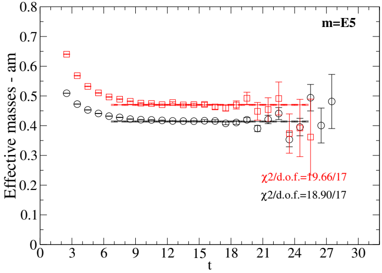

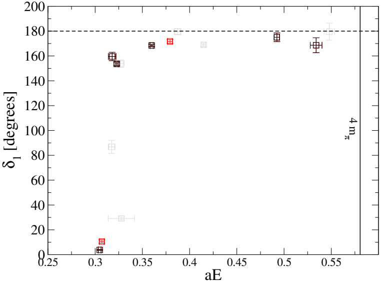

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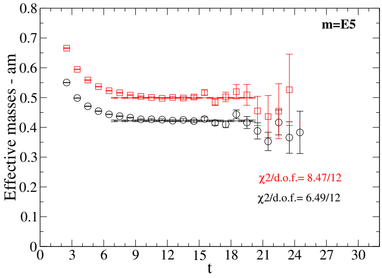

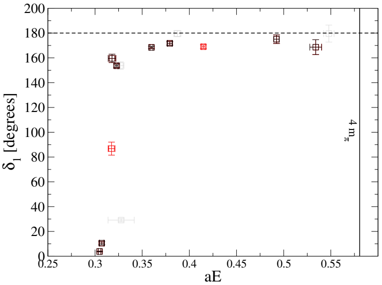

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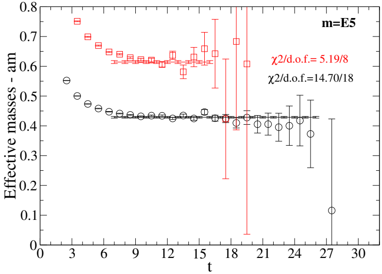

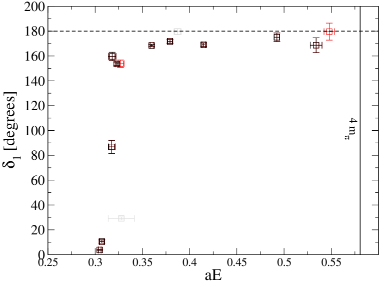

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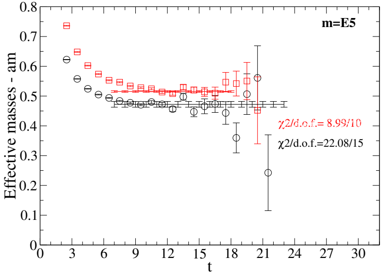
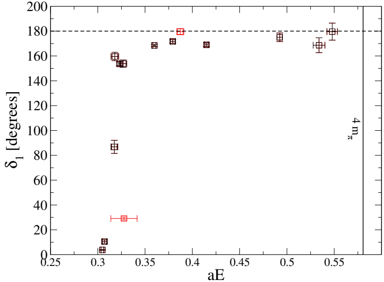
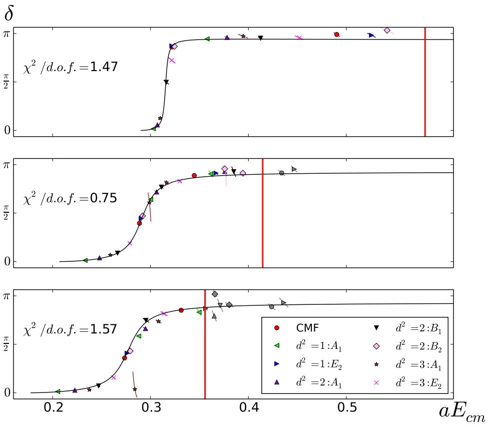




ρ resonance: Phase shift (preliminary)

F. Erben, DM *et al.* to be published



	E5	
	BW	GS
m_ρ	0.3156(8)	0.3157(10)
$g_{\rho\pi\pi}$	5.70(9)	5.66(9)
χ^2/dof	1.47	1.64
	F6	
	BW	GS
m_ρ	0.2933(8)	0.2934(9)
$g_{\rho\pi\pi}$	6.08(13)	6.03(13)
χ^2/dof	0.75	0.84
	F7	
	BW	GS
m_ρ	0.2800(11)	0.2800(10)
$g_{\rho\pi\pi}$	5.90(21)	5.87(17)
χ^2/dof	1.57	1.63

- Data above $4m_\pi$ (grey) not used
- Curve shows Breit-Wigner

The timelike pion form factor from LQCD

- Form factor in timelike region can be calculated as

H. B. Meyer, PRL 107,072002 (2011)

$$|(F_\pi)^{\mathbf{d}}_{\Lambda}(E)|^2 = G^{\mathbf{d}}_{\Lambda}(\gamma)\left(q(\phi^{\mathbf{d}}_{\Lambda})'(q) + k\frac{\partial\delta(k)}{\partial k}\right)\frac{3\pi E^2}{k^5}|A_\Psi|^2$$

- $|A_\Psi|^2 = |\langle\Omega|J(t)|n\rangle|^2$ from ratio of optimized two point functions

Andersen *et al.* NPB 939, 145, 2019

$$R_1(t) = \frac{\langle J(t)X_n^\dagger(0)\rangle}{\sqrt{D_{nn}(t)}e^{-\frac{1}{2}E_n t}} \rightarrow \frac{Z_n^*}{|Z_n|}\langle\Omega|J(t)|n\rangle$$

with

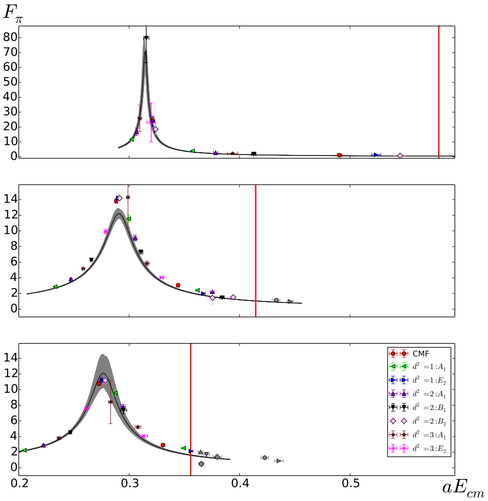
$$D_{nn}(t) = \langle X_n(t)X_n^\dagger(0)\rangle = v_n^\dagger C(t) v_n$$

$$\langle J(t)X_n^\dagger(0)\rangle = \sum_i v_{ni}\langle J(t)O_i^\dagger(0)\rangle$$

- We use both the local and conserved current (but no O(a) improvement)

Timelike pion form factor: Lattice results

F. Erben, DM *et al.* to be published



- Curves from Gounaris-Sakurai parameterization using determined $m_\rho, g_{\rho\pi\pi}$
- Qualitative agreement, but would like to fit the form factor

Fits to the data using Omnès representation

- n-subtracted Omnès representation

$$F(t) = \exp\left(P_{n-1}(t)t + \frac{t^n}{\pi} \int_{4m_\pi^2}^{\infty} ds \frac{\delta_{11}(s)}{s^n(s-t-i\epsilon)} \right)$$

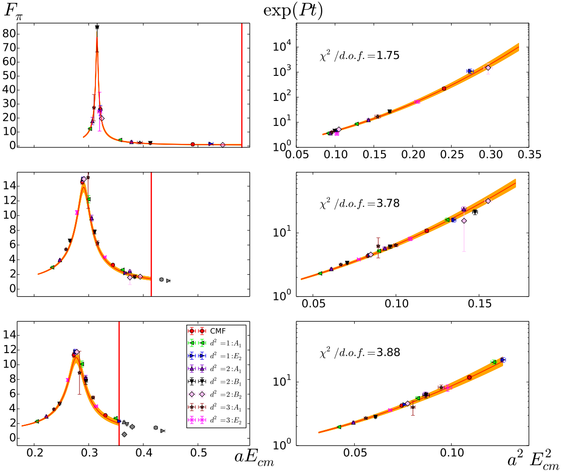
- We use terms for the square radius $\langle r^2 \rangle$ and curvature c_v^π

$$P(t) = \frac{\langle r^2 \rangle}{6} + \frac{1}{2}\left(2c_V^\pi - \left(\frac{\langle r^2 \rangle}{6}\right)^2\right)t$$

- To solve the integral we write

$$\int_{4m_\pi^2}^{\infty} ds \frac{\delta_{11}(s)}{s^n(s-t-i\epsilon)} = \int_{4m_\pi^2}^{\infty} ds \frac{\delta_{11}(s)-\delta_{11}(t)}{s^n(s-t)} + \delta_{11}(t) \int_{4m_\pi^2}^{\infty} ds \frac{1}{s^n(s-t-i\epsilon)}$$

F. Erben, DM *et al.* to be published



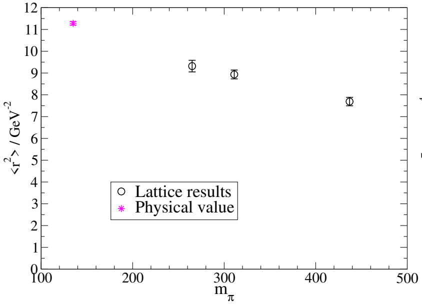
- Much better description of the lattice data
- Goodness of fit still not great
- Not caused by autocorrelation in Monte-Carlo chain

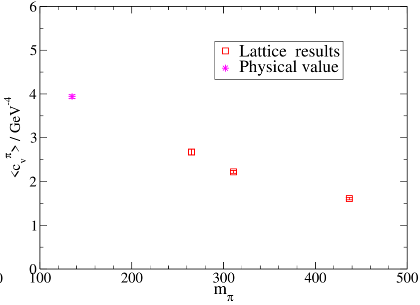
Comparison to spacelike results and phenomenology

- Comparison to spacelike results from `JHEP 1311 (2013) 034`

	E5	F6	F7
$\langle r^2\rangle/r_0^2$	1.18(3)	1.37(3)	1.43(4)
$c_V/r_0^4 \times 10^{-2}$	3.81(7)	5.26(8)	6.33(15)
$\langle r^2\rangle/r_0^2$	1.18(5)	1.37(6)	1.61(10)

- Local-conserved vs. local-local currents: significant discretization effects
- Good qualitative agreement with pion mass calculated by `Guo et. al. Phys.Lett. B678 (2009) 90-96`





Comparison to arXiv:1902.02273

Outline

Exotic D_s and B_s candidates

Established s and p-wave D_s and B_s hadrons:

D_s ($J^P = 0^-$) and D_s^* (1^-)	B_s ($J^P = 0^-$) and B_s^* (1^-)
$D_{s0}^*(2317)$ (0^+), $D_{s1}(2460)$ (1^+),	?
$D_{s1}(2536)$ (1^+), $D_{s2}^*(2573)$ (2^+)	$B_{s1}(5830)$ (1^+), $B_{s2}^*(5840)$ (2^+)

- Corresponding $D_0^*(2400)$ and $D_1(2430)$ are broad resonances
- Peculiarity: $M_{c\bar{s}} \approx M_{c\bar{d}} \quad \rightarrow \quad$ exotic structure? (tetraquark, molecule)

- B_s cousins of the $D_{s0}^*(2317)$ and $D_{s1}(2460)$ not (yet) seen in experiment
- The LHCb experiment at CERN should be able to see these
- Belle-II should be able to see these

$D^*_{s0}(2317)$: D-meson – Kaon s-wave scattering

M. Lüscher Commun. Math. Phys. 105 (1986) 153;
Nucl. Phys. B 354 (1991) 531; Nucl. Phys. B 364 (1991) 237.

Charm-light hadrons

$D^*_{s0}(2317)^\pm$

$I(J^P) = 0(0^+)$
J, P need confirmation.

J^P is natural, low mass consistent with 0^+.

Mass $m = 2317.7 \pm 0.6$ MeV
$m_{D^*_{s0}(2317)^\pm} - m_{D_s^\pm} = 349.4 \pm 0.6$ MeV
Full width $\Gamma < 3.8$ MeV, CL = 95%

$$p \cot \delta_0(p) = \frac{2}{\sqrt{\pi}L} Z_{00}\left(1; \left(\frac{L}{2\pi}p\right)^2\right)$$
$$\approx \frac{1}{a_0} + \frac{1}{2} r_0 p^2$$

Mohler *et al.* PRL 111 222001 (2013)
Lang, DM *et al.* PRD 90 034510 (2014)

Results for ensembles (1) and (2)

$$a_0 = -0.756 \pm 0.025\text{fm} \qquad (1)$$
$$r_0 = -0.056 \pm 0.031\text{fm}$$
$$a_0 = -1.33 \pm 0.20\text{fm} \qquad (2)$$
$$r_0 = 0.27 \pm 0.17\text{fm}$$

M. Lüscher Commun. Math. Phys. 105 (1986) 153;
Nucl. Phys. B 354 (1991) 531; Nucl. Phys. B 364 (1991) 237.

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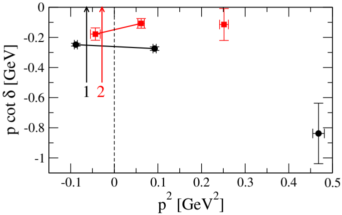

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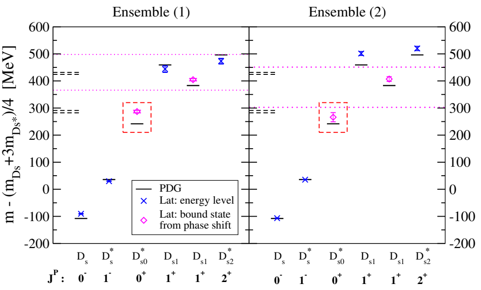
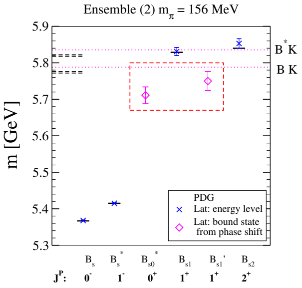
$$r_0 = 0.27 \pm 0.17\text{fm}$$

D_s and B_s: Spectrum results

Mohler *et al.* PRL 111 222001 (2013)
Lang, Mohler *et al.* PRD 90 034510 (2014)

Lang, Mohler, Prelovsek, Woloshyn PLB 750 17 (2015)



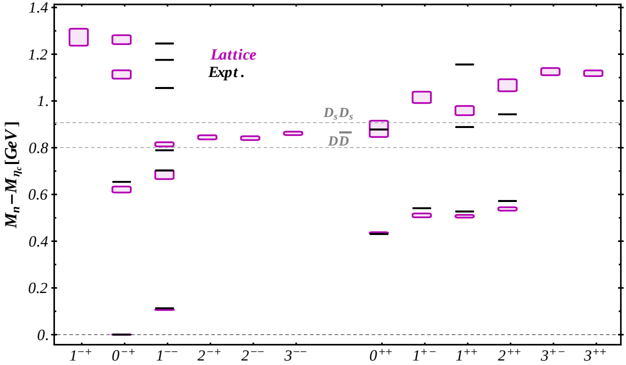


- Discretization uncertainties sizeable for charm
- Many improvements possible for the D_s states

- Full uncertainty estimate only for magenta B_s states
- Prediction of exotic states from Lattice QCD!

$\Psi(3770)$ and $X(3842)$ in $\bar{D}D$ scattering

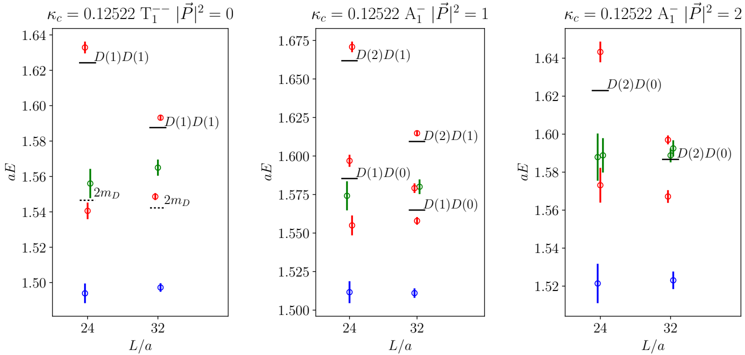
M. Padmanath *et al.*, PR**D** 99, 014513 (2019)



- Single hadron spectrum at $m_\pi = 280$ MeV
- Uses technique to identify continuum quantum numbers unambiguously
- Dashed lines indicate lowest open-charm thresholds on our lattices
- Will look at $\Psi(3770)$ and $X(3842)$ in $\bar{D}D$ scattering

Finite volume spectra with $\bar{q}q$ and $\bar{D}D$ interpolators

S. Piemonte, DM *et al.* to be published



- We only consider elastic $\bar{D}D$ scattering with $l = 1, 3$
- Color code
  - blue: energy level related to the $\psi(2S)$
  - green: energy level related to the presence of a spin 3 state
  - red: all other energy levels
- Clear finite volume energy shifts visible

Setup and parameterizations

S. Piemonte, DM *et al.* to be published

- We use two different charm-quark masses
- Single channel quantization condition (elastic scattering):

$$\det[\tilde{K}_l^{-1}(E_{cm})\,\delta_{l'l} - B_{l'l}^{\vec{P};\Lambda}(E_{cm})] = 0$$
$$\tilde{K}_l^{-1}(E_{cm}) = p^{2l+1}\cot\delta_l(p)$$

- Note that B is not diagonal in l
- Parameterizations used
  - $l = 1$

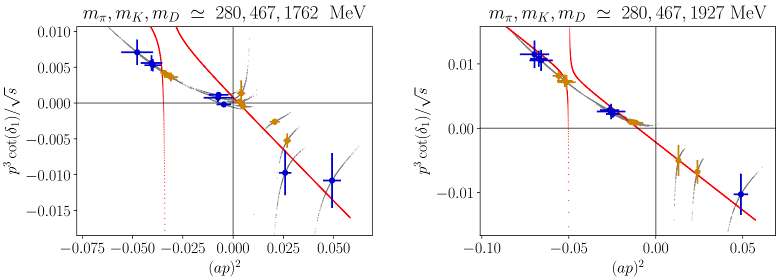
$$\frac{p^3\cot(\delta_1)}{\sqrt{s}} = \left(\frac{G_1^2}{m_1^2 - s} + \frac{G_2^2}{m_2^2 - s}\right)^{-1}$$
$$\frac{p^3\cot(\delta_1)}{\sqrt{s}} = A + Bs + Cs^2$$

  - $l = 3$

$$\frac{p^7\cot(\delta_3)}{\sqrt{s}} = \frac{m_3^2 - s}{g_3^2}$$

Results for $l = 1$ at two charm-quark masses

S. Piemonte, DM *et al.* to be published



- Color code:

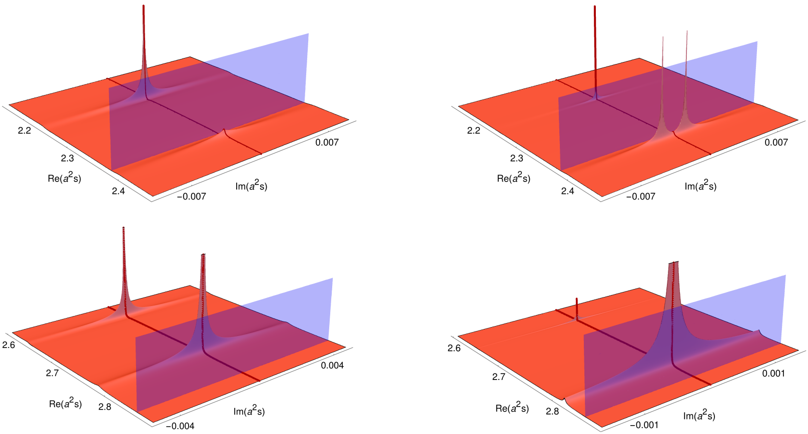
  blue: $L = 24$ orange: $L = 32$

- Naive expectation: Bound state for heavier charm-quark mass
- Naive expectation: Resonance for lighter charm-quark mass

Pole positions: amplitude modulus $|t_{l=1}|$

S. Piemonte, DM *et al.* to be published

Riemann sheet I | Riemann sheet II

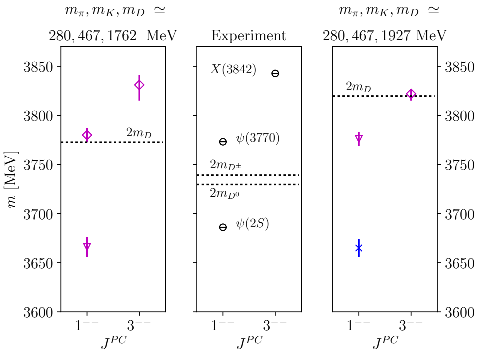


Final results I

	J^{PC}	present work $\kappa_c = 0.12522$	present work $\kappa_c = 0.12315$	exp $\bar{D}^0 D^0 / D^+ D^-$	arXiv:1503.05363
m_D [MeV]		1762(2)	1927(2)	$\bar{m}_D \simeq 1867$ MeV	1763(22)(18)*
m_{D_s} [MeV]		1818(1)	1981(1)	1968.34(7)	
M_{av} [MeV]		2820(3)	3103(3)	3068.6(2)	3119(9)(33)*
m_π [MeV]		280	280	$\bar{m}_\pi \simeq 137$ MeV	266
$\psi(3770)$	1^{--}	resonance	bound st.	resonance	resonance
g		$\mathbf{16.0(^{+2.1}_{-0.2})}$	$\mathbf{18.9(^{+0.8}_{-0.7})}$	**18.7(9)**	13.2(1.2)
$m - M_{av}$ [MeV]		711(7)	707(7)	704.25(35)	715(7)
$m - 2m_D$ [MeV]		9(7)	-43(8)	38.52(35)	
m [MeV]		**3780(7)**	**3776(7)**	**3773.13(35)**	3784(7)
$\psi(2S)$	1^{--}	bound st.	bound st.	bound st.	bound st.
$m - M_{av}$ [MeV]		597(10)	596(9)	617.347(25)	605(6)
$m - 2m_D$ [MeV]		-105(11)	-154(10)	-48.383(25)	
m [MeV]		**3666(10)**	**3665(9)**	**3686.097(25)**	3674(6)
$X(3842)$	3^{--}	resonance	resonance	resonance	
$m - M_{av}$ [MeV]		$762(^{+10}_{-16})$	$754(^{+4}_{-7})$	773.9(2)	
$m - 2m_D$ [MeV]		$59(^{+11}_{-16})$	$4(^{+9}_{-3})$	108.2(2)	
m [MeV]		$\mathbf{3831(^{+10}_{-16})}$	$\mathbf{3822(^{+4}_{-7})}$	**3842.7(2)**	

Final results II

S. Piemonte, DM *et al.* to be published



- Masses from $(2m_D - M_{\text{av}})^{\text{lat}} + (M_{\text{av}})^{\text{exp}}$
- Results are close to the ones seen in experiment
- Future: Chiral-continuum limit needs to be approached with care

χ'_{c0} and $X/Y(3915)$: A bit of history

$X(3915)$
was $\chi_{c0}(3915)$

$I^G(J^{PC}) = 0^+(0 \text{ or } 2^{++})$

Mass $m = 3918.4 \pm 1.9$ MeV
Full width $\Gamma = 20 \pm 5$ MeV (S = 1.1)

PDG interpreted $X(3915)$ as a regular charmonium (χ'_{c0})

- Some of the reasons to doubt this assignment:

Guo, Meissner Phys. Rev. **D**86, 091501 (2012)

Olsen, PRD 91 057501 (2015)

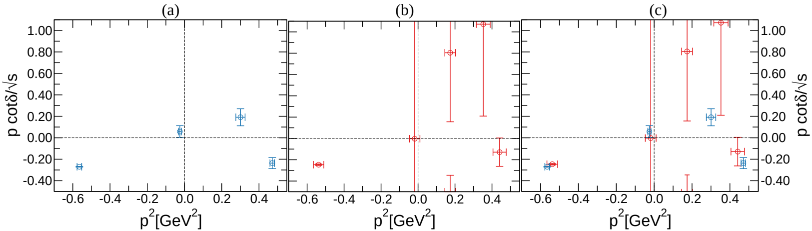
  - No evidence for fall-apart mode $X(3915) \to \bar{D}D$
  - Spin splitting $m_{\chi_{c2}(2P)} - m_{\chi_{c0}(2P)}$ too small
  - Large OZI suppressed $X(3915) \to \omega J/\psi$
  - Width should be significantly larger than $\Gamma_{\chi_{c2}(2P)}$
- Zhou *et al.* (PRL 115 2, 022001 (2015)) argue that what is dubbed X(3915) is the spin 2 state already known and suggests that a broader state is hiding in the experiment data.
- Observation of an alternative $\chi_{c0}(2P)$ by Belle:

Chilikin *et al.* PRD 95 112003 (2017)

$$M = 3862^{+26+40}_{-32-13}\text{ MeV} \qquad \Gamma = 201^{+154+88}_{-067-82}\text{ MeV}$$

χ'_{c0}: Exploratory lattice calculation

Lang, Leskovec, DM, Prelovsek, JHEP 1509 089 (2015)



- Assumes only $\bar{D}D$ is relevant
- Lattice data suggests a fairly narrow resonance with $3.9\text{GeV} < M < 4.0\text{GeV}$ and $\Gamma < 100\text{MeV}$
- Future experiment and lattice QCD results needed to clarify the situation

χ'_{c0}: Improvements and challenges

with S. Collins, M. Padmanath, S. Piemonte, S. Prelovsek

Improvements:

- High-precision determinations of the energy splittings needed
  $\rightarrow$ significantly improve statistics by using CLS ensembles
- Bigger density of energy level needed
  $\rightarrow$ Calculation in multiple volumes: CLS ensembles U101, H105, N101
  $\rightarrow$ Add information from moving frames
- Treatment as a single-channel problem only sensible if $X(3915)$ is indeed a spin-2 state
  $\rightarrow$ consider coupled channel $D\bar{D}$, $J/\psi\omega$ and $D_s\bar{D}_s$

(Specific) challenges:

- $Tr(M) =$ const. trajectory means $D_s\bar{D}_s$ threshold lower

Outlook

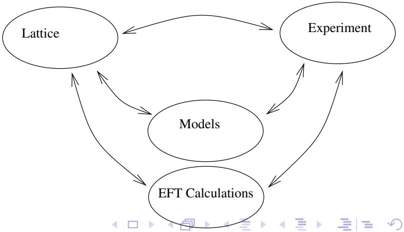
- Lattice calculations of scattering amplitudes are starting to mature
- Charmonium more difficult than light-quark and heavy-light mesons

Some powerful QCD tools:

- Can map out the quark mass dependence of amplitudes
  - heavy quark-mass dependence of a $X(3872)$ pole?
  - do bottom analogues of charm-quark states exist?
- Can investigate properties of short-lived excitations
- Can investigate states hard to produce/detect at current/future facilities

Possible strategy

- Calculate simple observables directly
- Test model predictions
- Use EFT results to relate to experiment



What about lattice results for $\overline{\text{P}}$anda/FAIR?

- Considerable opportunities for:
  - Low-lying strange baryons including higher spin
  - Low-lying charmed baryons including higher spin
  - Hyperon-Goldstone boson and hyperon-hyperon scattering at low energies
    $\rightarrow$ input for EFT calculations
  - Precision calculation of the $D_s^*(2317)$ including radiative transitions
  - Charmonium spectrum around the lowest (double) open-charm thresholds
- Novel methods/ideas needed:
  - Glueballs from full QCD (including mixing)
    $\rightarrow$ Is it reasonable that these are narrow?
  - Highly excited states (many open thresholds)
  - Charm annihilation contributions
- Move from exploratory calculations to comprehensive spectroscopy studies requires a dedicated commitment.
  Development has been rapid but should not be taken for granted!

Thank you!

Thanks to my colleagues in the Fermilab Lattice and MILC collaborations
Thanks to Felix Erben, Jeremy Green Hartmut Wittig
Thanks to M. Padmanath, Stefano Piemonte, Sara Collins, Sasa Prelovsek

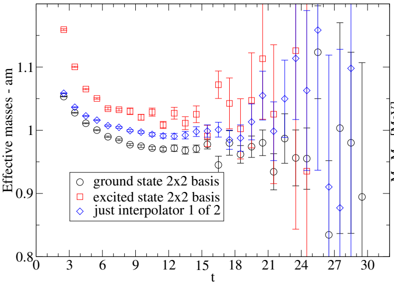
Further selected spectroscopy projects at HIM

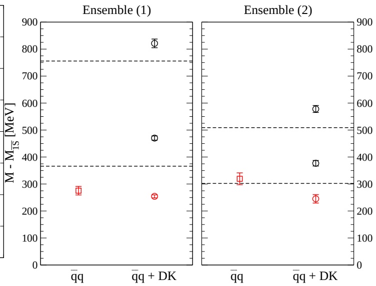
- π–K scattering with Isopin 1/2 and 3/2 in S- and P-wave
  - Determination of the S-wave scattering lengths
  - Properties of the κ ($K_0^*(700)$) and $K^*(892)$ resonances
  - Project together with Ruairi Brett, John Bulava, Andrew Hanlon, Ben Hörz, C. Morningstar
- Coupled channel N–π and Σ–$\bar{K}$ scattering and the $\Lambda(1405)$
  Project together with John Bulava, Ben Hörz, C. Morningstar
- The H-Dibarion from Lattice QCD
  Project by Andrew Hanlon, Parikshit Junnarkar, Hartmut Wittig
  See `Francis et al. arXiv:1805.03966`

A lesson about the interpolator basis

- “Fake” plateaus with an incomplete basis

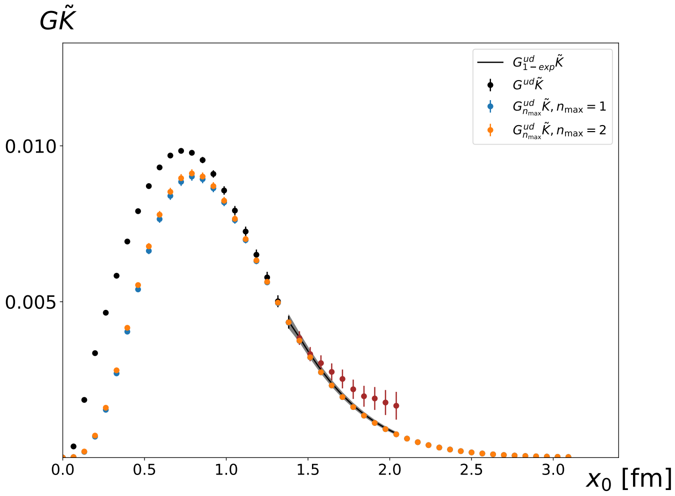
A diverse interpolator basis is vital to determine the true spectrum!





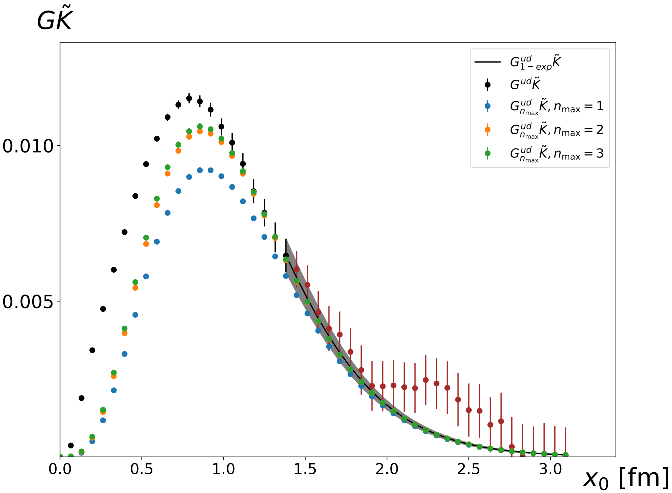
Data from Mohler *et al.* PRL 111 222001 (2013)

HVP – correlator reconstruction



- Reconstructing the integrand from GEVP energies and overlaps

HVP – correlator reconstruction



- Reconstructing the integrand from GEVP energies and overlaps

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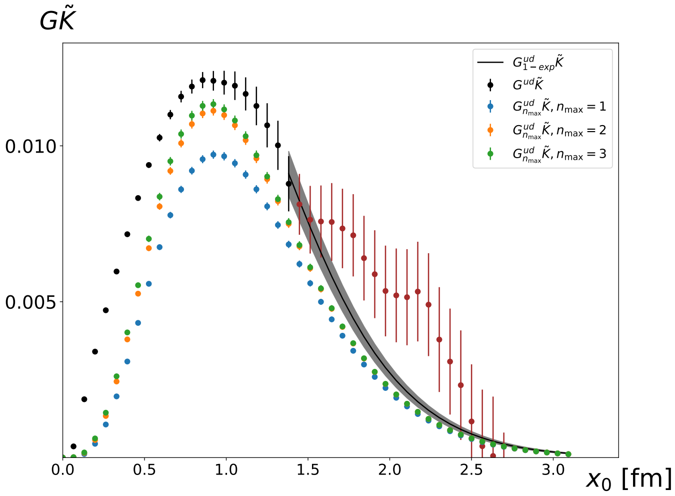
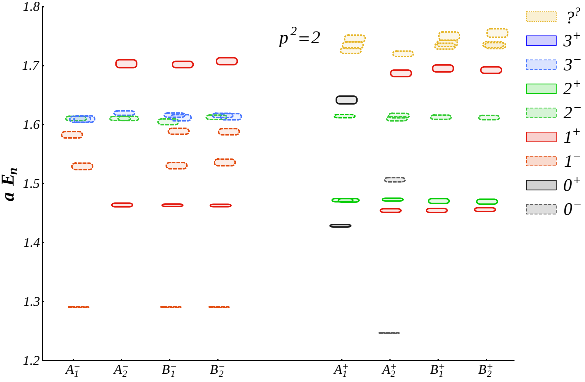


- Reconstructing the integrand from GEVP energies and overlaps

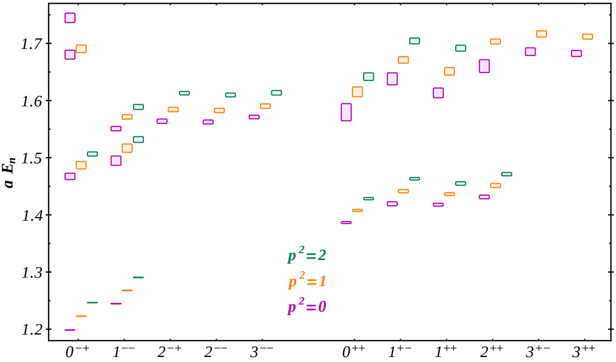
Example: J^P-identified spectrum with $p^2 = 2$

M. Padmanath *et al.*, PR**D** 99, 014513 (2019)



Charmonium energies for various momenta

M. Padmanath *et al.*, PR**D** 99, 014513 (2019)



- The same states are seen in the rest frame and in moving frames